## Official Nomenclature of US and European Societies of Rheology (2018-10)

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TABLE I. Steady simple shear (viscometric flow)

| Name | Definition | Symbol | SI units ${ }^{\text {a }}$ |
| :--- | :--- | :--- | :--- |
| Direction of flow (simple shear) |  | $x_{1}$ | m |
| Direction of velocity gradient (simple shear) |  | $x_{2}$ | m |
| Neutral direction (simple shear) |  | $x_{3}$ | m |
| Shear or normal force | $m a$ | $F$ | N |
| Velocity | $d x_{1} / d t$ | $v_{1}$ | $\mathrm{~m} \mathrm{~s}^{-1}$ |
| Acceleration | $d v_{1} / d t$ | $a_{1}$ | $\mathrm{~m} \mathrm{~s}^{-2}$ |
| Shear stress ${ }^{\mathrm{b}}$ | $F / A$ | $\sigma$ | Pa |
| Shear strain | $d x_{1} / d x_{2}$ | $\gamma$ | - |
| Shear rate | $\left\|d v_{1} / d x_{2}\right\|$ | $\dot{\gamma}$ | $\mathrm{s}^{-1}$ |
| Vorticity ${ }^{\text {c }}$ | $-d v_{1} / d x_{2}$ | $\omega_{3}$ | $\mathrm{~s}^{-1}$ |
| Viscosity | $\|\sigma\| / \dot{\gamma}$ | $\eta(\dot{\gamma})$ | Pa s |
| Yield stress |  | $\sigma_{y}$ | Pa |
| Yield strain |  | $\gamma_{y}$ | - |
| First normal stress difference | $\sigma_{11}-\sigma_{22}$ | $N_{1}$ | Pa |
| Second normal stress difference | $\sigma_{22}-\sigma_{33}$ | $N_{2}$ | Pa |
| First normal stress coefficient | $N_{1} / \dot{\gamma}^{2}$ | $\Psi_{1}$ | Pa s |
| Second normal stress coefficient | $N_{2} / \dot{\gamma}^{2}$ | $\Psi_{2}$ | Pa s |
| Normal stress ratio | $-N_{1} / N_{2}$ | $\Psi(\dot{\gamma})$ | - |
| Zero-shear viscosity <br> (limiting low shear rate viscosity) | $\eta(\dot{\gamma} \rightarrow 0)$ | $\eta_{0}$ | Pa s |
| Zero-shear first normal stress coefficient | $\Psi_{1}(\dot{\gamma} \rightarrow 0)$ | $\Psi_{1.0}$ | $\mathrm{~Pa} \mathrm{~s}^{2}$ |
| Consistency index in power law for viscosity | $\eta=K \dot{\gamma}^{n-1}$ | $n$ | - |
| Constant in power law for viscosity | see above equation | $K$ | $\mathrm{~Pa} \mathrm{~s}^{n}$ |

${ }^{\text {a }}$ SI allows either a dot between units or a space, as used here.
${ }^{\mathrm{b}} A$ is the area in $\mathrm{m}^{2}$.
${ }^{\mathrm{c}}$ In the fluid dynamics community a prefactor of $1 / 2$ is used in the definition of the vorticity.

TABLE II. Linear viscoelasticity

| Name | Definition | Symbol | Units |
| :---: | :---: | :---: | :---: |
| Simple shear |  |  |  |
| Shear modulus of a solid | $\sigma / \gamma$ | G | Pa |
| Relaxation modulus | $\sigma(t) / \gamma$ | $G(t)$ | Pa |
| Relaxation strength in discrete spectrum | - | $g_{i}$ | Pa |
| Relaxation time in discrete spectrum | - | $\tau_{i}$ | S |
| Continuous relaxation spectrum | $-{ }^{\text {a }}$ | $H(\tau)$ | Pa |
| Orthogonal superposition complex modulus ${ }^{\text {b,c }}$ | - | $G_{\perp}^{*}(\omega, \dot{\gamma})$ | Pa |
| Parallel superposition complex modulus ${ }^{\text {b,c }}$ | $-$ | $G_{\\|}^{*}(\omega, \dot{\gamma})$ | Pa |
| Memory function | $-d G(s) / d s$ | $m(s)$ | $\mathrm{Pa} \mathrm{s}^{-1}$ |
| Creep compliance (shear) | $\gamma(t) / \sigma$ | $J(t)$ | $\mathrm{Pa}^{-1}$ |
| Equilibrium compliance of solid | $J(t)(t \rightarrow \infty)$ | $J_{e}$ | $\mathrm{Pa}^{-1}$ |
| Recoverable compliance | $J(t)-t / \eta_{0}$ | $J_{r}(t)$ | $\mathrm{Pa}^{-1}$ |
| Steady-state compliance of fluid | $J(t)-t / \eta_{0}(t \rightarrow \infty)$ | $J_{S}^{0}$ | $\mathrm{Pa}^{-1}$ |
| Continuous retardation spectrum ${ }^{\text {d }}$ | - ${ }^{\text {d }}$ | $L(\tau)$ | $\mathrm{Pa}^{-1}$ |
| Small-amplitude oscillatory shear |  |  |  |
| Strain amplitude | $\gamma(t)=\gamma_{0} \sin \omega t$ | $\gamma_{0}$ | - |
| Loss angle (phase angle) | $\sigma(t)=\sigma_{0} \sin (\omega t+\delta)$ | $\delta$ | rad |
| Stress amplitude | $\sigma(t)=\sigma_{0} \sin (\omega t+\delta)$ | $\sigma_{0}$ | Pa |
| Complex modulus | $G^{\prime}+i G^{\prime \prime}$ | $G^{*}$ | Pa |
| Absolute magnitude of $G^{*}$ | $\sigma_{0} / \gamma_{0}$ | $\begin{aligned} & \left\|G^{*}\right\| \text { or } \\ & G_{d} \\ & \hline \end{aligned}$ | Pa |
| Storage modulus | $G_{d} \cos \delta$ | $G^{\prime}$ | Pa |
| Loss modulus | $G_{d} \sin \delta$ | $G^{\prime \prime}$ | Pa |
| Complex viscosity | $\eta^{\prime}-i \eta^{\prime \prime}$ | $\eta^{*}$ | Pa s |
| Absolute magnitude of $\eta^{*}$ | $\sigma_{0} / \omega \gamma_{0}$ | $\left\|\eta^{*}\right\|$ | Pas |
| Dynamic viscosity (in phase with strain rate) | $G^{\prime \prime} / \omega$ | $\eta^{\prime}$ | Pas |
| Out-of-phase (with strain rate) component of $\eta^{*}$ | $G^{\prime} / \omega$ | $\eta^{\prime \prime}$ | Pa s |
| Complex compliance | $J^{\prime}-i J^{\prime \prime}$ | $J^{*}$ | $\mathrm{Pa}^{-1}$ |
| Absolute magnitude of $J^{*}$ | $\gamma_{0} / \sigma_{0}=1 / G_{d}$ | $\left\|J^{*}\right\|$ | $\mathrm{Pa}^{-1}$ |
| Storage compliance | $\left(\gamma_{0} / \sigma_{0}\right) \cos \delta$ | $J^{\prime}$ | $\mathrm{Pa}^{-1}$ |
| Loss compliance | $\left(\gamma_{0} / \sigma_{0}\right) \sin \delta$ | $J^{\prime \prime}$ | $\mathrm{Pa}^{-1}$ |
| Plateau modulus ${ }^{\text {e }}$ | $-^{\text {e }}$ | $G_{N}^{0}$ | Pa |
| Tensile (Uniaxial) Extension ${ }^{\text {c }}$ |  |  |  |
| Net tensile stress | $\sigma_{z z}-\sigma_{r r}$ | $\sigma_{E}$ | Pa |
| Hencky strain | $\ln \left(L / L_{0}\right)$ | $\varepsilon$ | - |
| Hencky strain rate | $d(\ln L) / d t$ | $\varepsilon$ | $\mathrm{s}^{-1}$ |
| Tensile relaxation modulus | $\sigma_{E}(t) / \varepsilon_{0}$ | $E(t)$ | Pa |
| Tensile creep compliance | $\varepsilon_{0}(t) / \sigma_{E}$ | $D(t)$ | $\mathrm{Pa}^{-1}$ |

${ }^{\mathrm{a}} G(t)=\int_{-\infty}^{\infty} H(\tau)[\exp (-t / \tau)] d(\ln \tau)$
${ }^{\mathrm{b}}$ The same subscripts apply to definitions of storage and loss moduli and viscosities.
${ }^{\text {c }}$ Also applies to nonlinear phenomena in uniaxial extension.
${ }^{\mathrm{d}} J(t)=\int_{-\infty}^{\infty} L(\tau)[1-\exp (t / \tau)] d(\ln \tau)$
${ }^{\mathrm{e}}$ Because there is not a true plateau in $G(t)$ or $G^{\prime}(\omega), G_{N}^{0}$ is inferred from $G^{\prime}(\omega)$ and $G^{\prime \prime}(\omega)$ using methods reviewed by Liu et al. [Polymer 47, 4461-4479 (2006)].

TABLE III. Shift factors for time-temperature superposition

| Name | Definition | Symbol | SI Units |
| :--- | :--- | :--- | :--- |
| Vertical shift factor ${ }^{\text {a }}$ | $b_{T} G^{\prime}\left(T, \omega a_{T}\right)=G^{\prime}\left(T_{0}, \omega\right)$ | $b_{T}$ | - |
| Horizontal shift factor | $b_{T} G^{\prime}\left(T, \omega a_{T}\right)=G^{\prime}\left(T_{0}, \omega\right)$ | $a_{T}$ | - |
| First WLF coefficient | $\log _{10} a_{T}=\frac{-c_{1}\left(T-T_{0}\right)}{\left[c_{2}+\left(T-T_{0}\right)\right]}$ | $c_{1}$ | - |
| Second WLF coefficient | $\log _{10} a_{T}=\frac{-c_{1}\left(T-T_{0}\right)}{\left[c_{2}+\left(T-T_{0}\right)\right]}$ | $c_{2}$ | K |
| Activation energy for flow | $a_{T}=\exp \left[\frac{E_{a}}{R}\left(\frac{1}{T}-\frac{1}{T_{0}}\right)\right]$ | $E_{a}$ | $\mathrm{~kJ} \mathrm{~mol}^{-1}$ |

${ }^{\text {a }}$ For rubbers, $b_{\mathrm{T}}$ is given by $T_{0} \rho_{0} / T \rho$, where $\rho$ is mass density, with $\rho_{0}=\rho\left(T_{0}\right)$ [J. D. Ferry, Viscoelastic properties of polymers, $3^{\text {rd }}$ Ed., Wiley, NY, 1980]. It has also been found to be appropriate for polymers in general.

TABLE IV. Nonlinear viscoelasticity in shear

| Name | Definition | Symbol | SI Units |
| :---: | :---: | :---: | :---: |
| Stress relaxation (step strain) |  |  |  |
| Strain amplitude |  | $\gamma_{0}$ | - |
| Relaxation modulus (nonlinear) | $\sigma(t) / \gamma_{0}$ | $G\left(t, \gamma_{0}\right)$ | Pa |
| Damping function in shear | $G\left(t, \gamma_{0}\right) / G(t)$ | $h\left(\gamma_{0}\right)$ | - |
| First normal stress relaxation function |  | $N_{1}\left(t, \gamma_{0}\right)$ | Pa |
| Second normal stress relaxation function |  | $N_{2}\left(t, \gamma_{0}\right)$ | Pa |
| First normal stress relaxation coefficient | $N_{1}\left(t, \gamma_{0}\right) / \gamma_{0}^{2}$ | $\Psi_{1}\left(t, \gamma_{0}\right)$ | Pa |
| Second normal stress relaxation coefficient | $N_{2}\left(t, \gamma_{0}\right) / \gamma_{0}^{2}$ | $\Psi_{2}\left(t, \gamma_{0}\right)$ | Pa |
| Start-up of steady shear (at fixed shear rate) |  |  |  |
| Shear stress growth function |  | $\sigma^{+}(t, \dot{\gamma})$ | Pa |
| Shear stress growth coefficient | $\sigma^{+}(t, \dot{\gamma}) / \dot{\gamma}$ | $\eta^{+}(t, \dot{\gamma})$ | Pas |
| First normal stress growth function | $\sigma_{11}-\sigma_{22}$ | $N_{1}^{+}(t, \dot{\gamma})$ | Pa |
| First normal stress growth coefficient | $N_{1}^{+}(t, \dot{\gamma}) / \dot{\gamma}^{2}$ | $\Psi_{1}^{+}(t, \dot{\gamma})$ | $\mathrm{Pa} \mathrm{s}^{2}$ |
| Second normal stress growth function | $\sigma_{22}-\sigma_{33}$ | $N_{2}^{+}(t, \dot{\gamma})$ | Pa |
| Second normal stress growth coefficient | $N_{2}^{+}(t, \dot{\gamma}) / \dot{\gamma}^{2}$ | $\Psi_{2}^{+}(t, \dot{\gamma})$ | Pa s ${ }^{2}$ |
| Stress Ratio | $N_{1}(\dot{\gamma}) / \sigma(\dot{\gamma})$ | SR | - |
| Cessation of steady shear ( $\dot{\gamma}=0$ from $t=0)$ |  |  |  |
| Shear stress decay function |  | $\sigma^{-}(t, \dot{\gamma})$ | Pa |
| Shear stress decay coefficient | $\sigma^{-}(t, \dot{\gamma}) / \dot{\gamma}$ | $\eta^{-}(t, \dot{\gamma})$ | Pas |
| First normal stress decay function | $\sigma_{11}-\sigma_{22}$ | $N_{1}^{-}(t, \dot{\gamma})$ | Pa |
| First normal stress decay coefficient | $N_{1}^{-}(t, \dot{\gamma}) / \dot{\gamma}^{2}$ | $\Psi_{1}^{-}(t, \dot{\gamma})$ | $\mathrm{Pa} \mathrm{s}{ }^{2}$ |
| Second normal stress decay function | $\sigma_{22}-\sigma_{33}$ | $N_{2}^{-}(t, \dot{\gamma})$ | Pa |
| Second normal stress decay coefficient | $N_{2}^{-}(t, \dot{\gamma}) / \dot{\gamma}^{2}$ | $\Psi_{2}^{-}(t, \dot{\gamma})$ | Pa s ${ }^{2}$ |
| Creep and creep recovery (recoil) |  |  |  |
| Creep compliance | $\gamma(t, \sigma) / \sigma$ | $J(t, \sigma)$ | $\mathrm{Pa}^{-1}$ |
| Steady-state compliance ${ }^{\text {a }}$ | $J(t \rightarrow \infty, \sigma)$ | $J_{s}(\sigma)$ | $\mathrm{Pa}^{-1}$ |
| Recoverable strain (after $t_{0}$ when $\sigma \rightarrow 0$ ) | $\begin{gathered} \gamma\left[t_{0}, \sigma\right]-\gamma[t, \sigma] \\ t>t_{0} \end{gathered}$ | $\begin{aligned} & \gamma_{r}\left(t^{\prime}, \sigma\right) \\ & t^{\prime}=t-t_{0} \end{aligned}$ | - |
| Ultimate recoil | $\gamma_{r}\left(t^{\prime} \rightarrow \infty, \sigma\right)$ | $\gamma_{\infty}(\sigma)$ | - |
| Steady-state recoverable compliance ${ }^{\text {a }}$ | $\gamma_{\infty}(\sigma) / \sigma$ | $J_{r}(\sigma)$ | $\mathrm{Pa}^{-1}$ |

${ }^{\text {a }}$ Although measured in different ways, the steady-state compliance and the steady-state recoverable compliance should be equal to each other according to the Boltzmann principle.

TABLE V. Nonlinear viscoelasticity in extension.

| Name | Definition | Symbol | SI Units |
| :--- | :--- | :--- | :--- |
| Tensile (uniaxial) extension |  |  | $\left(L-L_{0}\right) / L_{0}$ |
|  |  |  |  |
| Engineering strain ${ }^{\text {a }}$ | $\varepsilon$ | - |  |
| Engineering stress ${ }^{\text {a }}$ | $F / A_{0}$ | $\sigma$ | Pa |
| Young's modulus of a solid | $\sigma / \varepsilon$ | $E$ | Pa |
| Net tensile stress (true) | $\sigma_{z z}-\sigma_{r r}$ | $\sigma_{\varepsilon}$ | Pa |
| Hencky strain | $\ln \left(L / L_{0}\right)$ | $\varepsilon$ or $\epsilon$ | - |
| Hencky strain rate | $d(\ln L) / d t$ | $\dot{\varepsilon}$ or $\dot{\epsilon}$ | $\mathrm{s}^{-1}$ |
| Tensile stress growth function |  | $\sigma_{E}^{+}(t, \dot{\varepsilon})$ | Pa |
| Tensile stress growth coefficient | $\sigma_{E}^{+}(t, \dot{\varepsilon}) / \dot{\varepsilon}$ | $\eta_{E}^{+}(t, \dot{\varepsilon})$ | Pa s |
| Extensional viscosity | $\eta_{E}^{+}(t, \dot{\varepsilon}) t \rightarrow \infty$ | $\eta_{E}(\dot{\varepsilon})$ | Pa s |
| Tensile creep compliance | $\varepsilon(t) / \sigma_{E}$ | $D\left(t, \sigma_{E}\right)$ | Pa |
| Recoverable strain <br> (after $t_{0}$ where $\left.\sigma_{\mathrm{E}} \rightarrow 0\right)$ | $\varepsilon\left[t_{0}, \sigma_{E}\right]-\varepsilon\left[t, \sigma_{E}\right]$ | $\varepsilon_{r}\left(t^{\prime}, \varepsilon\right)$ <br> $t^{\prime}=t-t_{0}$ | - |
| Biaxial extension (symmetrical) |  | $t>t_{0}$ |  |
| Biaxial strain | $\ln \left(R / R_{0}\right)$ | $\varepsilon_{B}$ | - |
| Biaxial strain rate | $d(\ln R) / d t$ | $\dot{\varepsilon}_{B}$ | $\mathrm{~s}^{-1}$ |
| Net stretching stress | $\sigma_{r r}-\sigma_{z z}$ | $\sigma_{B}$ | Pa |
| Biaxial stretch growth function |  | $\sigma_{B}^{+}\left(t, \dot{\varepsilon}_{B}\right)$ | Pa |
| Biaxial stretch growth coefficient | $\sigma_{B}^{+}\left(t, \dot{\varepsilon}_{B}\right) / \dot{\varepsilon}_{B}$ | $\eta_{B}^{+}\left(t, \dot{\varepsilon}_{B}\right)$ | Pa s |
| Biaxial stress decay coefficient | $\sigma_{B}^{-\left(t, \dot{\varepsilon}_{B}\right) / \dot{\varepsilon}_{B}}$ | $\eta_{B}^{-}\left(t, \dot{\varepsilon}_{B}\right)$ | Pa s |
| Biaxial extensional viscosity | $\sigma_{B}^{+}\left(t \rightarrow \infty, \dot{\varepsilon}_{B}\right) / \dot{\varepsilon}_{B}$ | $\eta_{B}\left(\dot{\varepsilon}_{B}\right)$ | Pa s |
| Biaxial creep compliance | $\varepsilon_{B}(t) / \sigma_{B}$ | $D\left(t, \sigma_{B}\right)$ | $\mathrm{Pa}{ }^{-1}$ |

${ }^{a}$ In the mechanics literature, the same symbols are often used for both engineering and true stress and strain, but they are only equivalent in the limit of very small deformations.

## TABLE VI. Rheometry

| Name | Definition | Symbol | SI Units |  |
| :--- | :--- | :--- | :--- | :---: |
| Capillary rheometers |  |  |  |  |
|  |  |  |  |  |
| Apparent wall shear stress ${ }^{\mathrm{a}}$ | $P_{d} R / 2 L$ | $\sigma_{A}$ | Pa |  |
| Apparent wall shear rate | $4 Q / \pi R^{3}$ | $\dot{\gamma}_{A}$ | $\mathrm{~s}^{-1}$ |  |
| Wall shear stress | $-\sigma_{r Z}(r=R)$ | $\sigma_{W}$ | Pa |  |
| Wall shear rate | $-d v_{z} / d r(r=R)$ | $\dot{\gamma}_{W}$ | $\mathrm{~s}^{-1}$ |  |
| Cone-plate rheometers |  |  |  |  |
| Cone angle | Figure 1 | $\beta$ | rad |  |
| Angular rotation | Figure 1 | $\varphi$ | rad |  |
| Angular velocity | $d \varphi / d t$ | $\Omega$ | $\mathrm{rad} \mathrm{s}^{-1}$ |  |
| Torque ${ }^{\mathrm{b}}$ | $2 \pi R^{3} \sigma_{\varphi \theta} / 3$ | $M$ | N m |  |
| Normal thrust | Figure 1 | $F_{Z}$ | N |  |

${ }^{\text {a }} P_{d}=$ driving (reservoir) pressure (exit pressure neglected)
${ }^{\mathrm{b}}$ Approximation valid for $\beta<0.1 \mathrm{rad}$.


Figure 1. Symbols describing cone-plate geometry.

## TABLE VII. Solutions

| Name | Definition | Symbol | SI units |
| :--- | :--- | :--- | :--- |
| Concentration |  | $c$ | $\mathrm{~kg} \mathrm{~m}^{-3 \mathrm{a}}$ |
| Overlap <br> concentration |  | $c^{*}$ | $\mathrm{~kg} \mathrm{~m}^{3 \mathrm{a}}$ |
| Solvent viscosity |  | $\eta_{s}$ | Pa s |
| Relative viscosity | $\left(\eta / \eta_{s}\right)$ | $\eta_{r}$ | - |
| Specific viscosity | $\left(\eta_{r}-1\right)$ | $\eta_{s p}$ | - |
| Reduced viscosity | $\eta_{s p} / c$ | $\eta_{r e d}$ | $\mathrm{~m}^{3} \mathrm{~kg}^{-1 \mathrm{~b}}$ |
| Intrinsic viscosity | $\lim \left(\eta_{\mathrm{red}}, \dot{\gamma} \rightarrow 0, c \rightarrow 0\right)$ | $[\eta]$ | $\mathrm{m}^{3} \mathrm{~kg}^{-1 \mathrm{~b}}$ |
| Viscosity of matrix ${ }^{\mathrm{c}}$ |  | $\eta_{m}$ | Pa s |
| Solvent contribution <br> to the stress tensor |  | $\boldsymbol{\sigma}^{s}$ | Pa |
| Polymeric <br> contribution to the <br> stress tensor |  | $\boldsymbol{\sigma}^{p}$ | Pa |

${ }^{\text {a }}$ Units of $\mathrm{g} \mathrm{mL}{ }^{-1}$ are often used.
${ }^{\mathrm{b}}$ Units of $\mathrm{mL} \mathrm{g}^{-1}$ are often used.
${ }^{\mathrm{c}}$ Often used for nanocomposites, i.e., particles in a viscoelastic matrix.

## TABLE VIII. Suspensions

| Name | Definition | $\begin{array}{\|l} \hline \begin{array}{l} \text { Symb } \\ \text { ol } \end{array} \\ \hline \end{array}$ | $\begin{array}{\|l\|} \hline \text { SI } \\ \text { Units } \end{array}$ |
| :---: | :---: | :---: | :---: |
| Volume fraction solid | $V_{\text {solid }} / V_{\text {suspension }}$ | $\phi$ | - |
| Maximum packing fraction |  | $\phi_{\max }$ | - |
| Suspending medium viscosity ${ }^{\text {a }}$ |  | $\eta_{m}$ | Pas |
| Effective viscosity of suspension | $\Sigma_{12} / \dot{\gamma}$ | $\eta$ | Pas |
| Relative viscosity of suspension | $\eta / \eta_{f}$ | $\eta_{r}$ | - |
| Particle contribution to $\eta$ |  | $\eta_{p}$ | Pa S |
| Local stress tensor |  | $\boldsymbol{\sigma}(\boldsymbol{x}, t)$ | Pa |
| Total bulk stress | $\mathbf{\Sigma}^{f}+\mathbf{\Sigma}^{p}$ | $\Sigma$ | Pa |
| Fluid (suspending medium) contribution |  | $\Sigma^{f}$ | Pa |
| Particle contribution |  | $\Sigma^{p}$ | Pa |
| First normal stress difference | $\Sigma_{11}-\Sigma_{22}$ | $N_{1}$ | Pa |
| Second normal stress difference | $\Sigma_{22}-\Sigma_{33}$ | $N_{2}$ | Pa |
| Dimensionless first normal stress difference | $-N_{1} / \eta_{f} \dot{\gamma}$ | $\mathrm{r}_{1}$ | - |
| Dimensionless second normal stress difference | $-N_{2} / \eta_{f} \dot{\gamma}$ | $r_{2}$ | - |
| Particle pressure | $-\frac{1}{3}\left(\Sigma_{11}^{p}+\Sigma_{22}^{p}+\Sigma_{33}^{p}\right)$ | $\Pi$ | Pa |
| Hydrodynamic particle stress |  | $\Sigma^{H}$ | Pa |
| Interparticle stress |  | $\Sigma^{I P}$ | Pa |
| Brownian stress |  | $\Sigma^{B}$ | Pa |

${ }^{\text {a }}$ Sometimes the subscript $f$ is used to indicate the fluid viscosity, but it seems more adequate to use $m$ which stands for "medium" if the suspending fluid is itself rheologically complex or "matrix" in nanocomposites. Also, $\mu$ is often used to denote fluid viscosity.

## TABLE IX. Interfacial and surface rheology

| Name | Definition | Symbol | SI Units |
| :---: | :---: | :---: | :---: |
| Interfacial or surface tension ${ }^{\text {a }}$ |  | $\sigma_{\alpha, \beta}$ | Pam |
| Surface pressure | $\sigma_{\alpha, \beta}^{0}-\sigma_{\alpha, \beta}\left(F^{s} / A\right)$ | $\Pi$ | Pam |
| Interfacial shear stress | $F^{s} / L$ | $\sigma^{s}$ | Pam |
| Interfacial shear strain | $d x_{1} / d x_{2}$ | $\gamma^{s}$ | - |
| Interfacial shear rate | $d v_{1} / d x_{2}$ | $\dot{\gamma}^{s}$ | $\mathrm{s}^{-1}$ |
| Interfacial dilatational strain | $\ln \left(A / A_{0}\right)$ | $\alpha^{s}$ | - |
| Interfacial dilatational strain rate | $d(\ln A) / d t$ | $\dot{\alpha}^{s}$ | $\mathrm{s}^{-1}$ |
| Interfacial concentration | - | $\Gamma$ | $\mathrm{kg} \mathrm{m}^{-2}$ |
| Steady shear and dilatation |  |  |  |
| Interfacial shear viscosity | $\sigma^{s} / \dot{\gamma}^{s}$ | $\eta^{s}$ | Pasm |
| Interfacial dilatational viscosity | $\sigma^{s} / \dot{\alpha}^{s}$ | $\kappa^{s}$ | Pasm |
| Simple shear |  |  |  |
| Interfacial shear modulus | $\sigma^{s} / \gamma^{s}$ | $G^{s}$ | Pam |
| Relaxation modulus (shear) | $\sigma^{s}(t) / \gamma^{s}$ | $G^{s}(t)$ | Pam |


| Pure dilatation |  |  |  |
| :---: | :---: | :---: | :---: |
| Interfacial dilatational modulus | $\sigma^{s} / \alpha^{s}$ | $K^{s}$ | Pa m |
| Dilatational storage modulus | $\sigma^{s}(t) / \alpha^{s}$ | $K^{s}(t)$ | Pam |
| Gibbs elasticity (surfactants) ${ }^{\text {b }}$ | $d \sigma_{\alpha, \beta} / d(\ln A)$ | $K_{\text {П }}$ | Pa m |
| Small-amplitude oscillatory shear |  |  |  |
| Strain amplitude | $\gamma^{s}=\gamma_{0}^{s} \sin (\omega t)$ | $\gamma_{0}^{s}$ | - |
| Phase angle (loss angle) | $\sigma^{s}(t)=\sigma_{0}^{s} \sin (\omega t+\delta)$ | $\delta$ | rad |
| Stress amplitude | $F^{s} / L$ | $\sigma_{0}^{s}$ | Pam |
| Complex interfacial shear modulus | $G^{s \prime}+i G^{\text {s' }}$ | $G^{s *}$ | Pam |
| Absolute magnitude of $G^{\text {s* }}$ | $\sigma_{0}^{s} / \gamma_{0}^{s}$ | $\left\|G^{s *}\right\|$ | Pam |
| Storage modulus | $\left\|G^{S *}\right\| \cos \delta$ | $G^{\text {s' }}$ | Pam |
| Loss modulus | $\left\|G^{s *}\right\| \sin \delta$ | $G^{\text {st }}$ | Pam |
| Complex shear viscosity | $\eta^{s \prime}-i \eta^{s \prime \prime}$ | $\eta^{s *}$ | Pa sm |
| Absolute magnitude of $\eta^{s *}$ | $\sigma_{0}^{s} / \omega \gamma_{0}^{s}$ | $\left\|\eta^{s *}\right\|$ | Pa s m |
| Dynamic shear viscosity (in phase with strain rate) | $G^{s \prime \prime} / \omega$ | $\eta^{s \prime}$ | Pasm |
| Out-of-phase (with strain rate) component of $\eta^{s *}$ | $G^{s \prime} / \omega$ | $\eta^{s \prime \prime}$ | Pa sm |
| Small-amplitude oscillatory dilatation |  |  |  |
| Dilatational strain amplitude | $\alpha^{s}=\alpha_{0}^{s} \sin (\omega t)$ | $\alpha_{0}^{S}$ | - |
| Complex dilatational modulus ${ }^{\text {b }}$ | $K^{s^{\prime}}+i K^{s \prime \prime}$ | $K^{s *}$ | Pam |
| Absolute magnitude of $K^{s *} \mathrm{~b}$ | $\sigma_{0}^{S} / \alpha_{0}^{S}$ | $\left\|K^{s *}\right\|$ | Pam |
| Storage dilatational modulus ${ }^{\text {b }}$ | $K^{\text {S }}$ | $K^{\text {S }}$ | Pam |
| Loss dilatational modulus ${ }^{\text {b }}$ | $K^{\text {s }}$ | $K^{s \prime \prime}$ | Pam |
| Complex dilatational viscosity | $\kappa^{s^{\prime}}+i \kappa^{s^{\prime \prime}}$ | $\kappa^{s *}$ | Pa sm |
| Absolute magnitude of $\kappa^{s *}$ | $\sigma_{0}^{S} / \omega \delta_{0}^{S}$ | $\left\|\kappa^{s *}\right\|$ | Pasm |
| Dynamic dilatational viscosity | $K^{s \prime \prime} / \omega$ | $\kappa^{s \prime}$ | Pa sm |
| Out-of-phase component of $\kappa^{\text {S* }}$ | $K^{s \prime} / \omega$ | $\kappa^{\text {s/ }}$ | Pa s m |
| Other properties |  |  |  |
| Creep compliance (shear) | $\gamma^{s}(t) / \sigma^{s}$ | $J^{s}(t)$ | $\mathrm{Pa}^{-1} \mathrm{~m}^{-1}$ |
| Equilibrium compliance of solid | $J^{s}(t)(t \rightarrow \infty)$ | $J_{e}^{s}$ | $\mathrm{Pa}^{-1} \mathrm{~m}^{-1}$ |
| Recoverable compliance | $J^{s}(t)-t / \eta_{0}^{s}$ | $J_{r}^{s}(t)$ | $\mathrm{Pa}^{-1} \mathrm{~m}^{-1}$ |
| Steady-state compliance of fluid | $J^{s}(t)-t / \eta_{0}^{s}(t \rightarrow \infty)$ | $J_{0}^{S}$ | $\mathrm{Pa}^{-1} \mathrm{~m}^{-1}$ |
| Extensional viscosity | $\eta_{E}^{s}\left(t, \dot{\varepsilon}^{s}\right)(t \rightarrow \infty)$ | $\eta_{E}^{S}\left(\dot{\varepsilon}^{s}\right)$ | Pa s m |
| ${ }^{\text {a }}$ In some cases the symbols $\alpha$ or $\Gamma$ are used to denote surface tension (not to be confused with interfacial dilatational strain or surface concentration, respectively). |  |  | ed with oung's |

## TABLE X. Molecular description of entangled polymers

a tube diameter (typically in nm); average entanglement spacing / mesh size $\sqrt{\left\langle R^{2}\right\rangle_{0} M_{e} / M}$
$b_{\mathrm{K}} \quad$ Kuhn segment length ${ }^{\text {a }}$
$f \quad$ tension in a chain segment (typically in N )
$f_{\text {max }}$ maximum tension in a chain segment
$k_{\mathrm{B}} \quad$ Boltzmann's constant, $1.38 \times 10^{-23} \mathrm{~J} \mathrm{~K}^{-1}$
$L$ mean tube contour length
$M$ molecular weight ${ }^{\mathrm{b}}, \mathrm{kg} \mathrm{mol}^{-1}$
$M_{\mathrm{C}} \quad$ critical molecular weight for effect of entanglements on $\eta_{0}, \mathrm{~kg} \mathrm{~mol}^{-1}$
$M_{\mathrm{C}}^{\prime} \quad$ molecular weight for effect of entanglements on $J_{s}^{0}, \mathrm{~kg} \mathrm{~mol}^{-1}$
$M_{\mathrm{e}} \quad$ molecular weight between entanglements ${ }^{\mathrm{c}}\left(\rho R T / G_{N}^{0}\right)$
$N_{\mathrm{K}} \quad$ number of Kuhn segments in equivalent freely jointed chain ${ }^{\mathrm{a}}$
$p \quad$ packing length ${ }^{\text {d }}\left(M /\left[\left\langle R^{2}\right\rangle_{0} \rho N_{\mathrm{A}}\right]\right)$
$p^{2}$ parameter to denote degree of branch point hopping within tube (fraction of tube diameter)
$R \quad$ end-to-end distance of polymer molecule
$R_{\text {max }}$ fully extended chain length
$s \quad$ tube contour variable (curvilinear coordinate along tube)
$\boldsymbol{S}$ tube orientation tensor
$Z \quad$ number of entanglement segments per molecule ${ }^{\mathrm{e}}\left(M / M_{\mathrm{e}}\right)$

## Greek letters

$\alpha \quad$ dilution exponent for $M_{e}$
$\zeta \quad$ friction coefficient
$\zeta_{0} \quad$ monomer friction coefficient
$\lambda \quad$ chain stretch; stretch ratio
$\xi \quad$ correlation length; characteristic size scale (blob)
$\tau_{\mathrm{d}} \quad$ reptation (tube disengagement) time
$\tau_{\mathrm{e}} \quad$ Rouse time of an entanglement strand ( $\tau_{\mathrm{R}} / 3 \pi^{2} Z^{2}$ )
$\tau_{\mathrm{p}} \quad$ relaxation time of the $p^{\text {th }}$ mode ( $p$ is the mode index)
$\tau_{\mathrm{r}} \quad$ Rouse reorientation relaxation time
$\tau_{\mathrm{R}} \quad$ Rouse stress relaxation time $\left[\left(\zeta N^{2} b^{2}\right) /\left(6 \pi^{2} k_{\mathrm{B}} T\right)\right]$
$\tau_{\mathrm{s}} \quad$ stretch relaxation time ${ }^{\mathrm{f}}$
${ }^{\mathrm{a}} b_{\mathrm{K}}$ and $N_{\mathrm{K}}$ are defined by the following relationships: $\left\langle R^{2}\right\rangle=b_{\mathrm{K}}^{2} N_{\mathrm{K}} ; R_{\max }=b_{\mathrm{K}} N_{\mathrm{K}}$
${ }^{\mathrm{b}}$ IUPAC recommends molar mass (MM), which has SI units of $\mathrm{g} \mathrm{mol}^{-1}$. But molecular weight (MW) is widely used, and ACS accepts both terms. However, MW is formally a dimensionless ratio that is numerically very close to $\mathrm{MM}\left(\mathrm{g} \mathrm{mol}^{-1}\right)$, and one cannot "change its units". The number often called "molecular weight $\left(\mathrm{kg} \mathrm{mol}^{-1}\right)$ " is formally MW/1000 (no units). This quantity should more properly be called molar mass with units of $\mathrm{kg} \mathrm{mol}^{-1}$. But it is so widely called molecular weight that it seems hopeless to try to change it now.
${ }^{\mathrm{c}}$ This is the definition originally proposed by John Ferry. The following alternative definition was introduced much later in the context of Doi-Edwards model. It should be used in the context of the development of, or comparisons with tube-model theories: $M_{e}=\frac{4}{5} \rho R T / G_{N}^{0}$.
${ }^{\mathrm{d}}$ For a discussion of $p$ see Fetters et al. [Macromolecules 27, 4639 (1994)].
${ }^{\mathrm{e}}$ In tube-model theory, $Z$ is called "number of tube segments" and is defined based on the $M_{e}$ of footnote (c), i.e., using the Doi-Edwards $\frac{4}{5}$ factor. Hence, when comparing with or using tube models, the $\frac{4}{5}$ factor should be used.
${ }^{\mathrm{f}}$ The Rouse reorientation relaxation time was called the rotational relaxation time by Doi and Edwards and is conjectured to be equal to $2 \tau_{\mathrm{R}}$.

## Table XI. Stress and strain tensors

Total stress tensor $\boldsymbol{\sigma}$
Extra stress tensor $\boldsymbol{\tau}$
Strain tensor for linear viscoelasticity $\quad \boldsymbol{\gamma}$
Cauchy tensor
Finger tensor
Doi-Edwards strain tensor
Rate-of-strain tensor ${ }^{\text {a }}$
C

Vorticity tensor $\quad \boldsymbol{\Omega}=\boldsymbol{\nabla} \mathbf{v}-(\boldsymbol{\nabla} \mathbf{v})^{T}$
${ }^{\text {a }}$ An alternative definition, equal to $\frac{1}{2} \dot{\boldsymbol{\gamma}}$, is widely used in fluid mechanics and is acceptable, but the symbol $\boldsymbol{D}$ should be used for this tensor to avoid confusion: $\boldsymbol{D} \equiv \frac{1}{2}\left[\boldsymbol{\nabla} \mathbf{v}+(\boldsymbol{\nabla} \mathbf{v})^{T}\right]$, and in the fluid dynamics community a factor of $\frac{1}{2}$ is also used as a prefactor in the defintion of the vorticity tensor $\boldsymbol{\Omega}$.

Table XII. Dimensionless groups used to describe experimental regimes

Bingham Number ${ }^{\text {a }}$
Boussinesq Number
Capillary number

Péclet Number
Poisson ratio ${ }^{d}$
Reynolds number
Weissenberg Number be,f

Deborah Number ${ }^{\mathrm{b}, \mathrm{c}} \quad D e \equiv$ (characteristic time of fluid) / (duration of deformation)
[e.g., $D e=\tau \omega$ ]
$B n \equiv$ (yield stress) / (shear stress)
$B q \equiv($ surface shear stress $) /[($ bulk subphase shear stress $) \times$ (perimeter length along which the surface shear stress acts)]
$C a=\eta v / \sigma_{\alpha, \beta} \equiv$ (viscous forces) $/$ (surface forces)
or $C a=\eta \dot{\gamma} R / \sigma_{\alpha, \beta}$

Pé $=\dot{\gamma} a^{2} / D_{o} \equiv$ (rate of advection) / (rate of diffusion)
[ $a=$ particle radius, $D_{o}=$ particle diffusion coefficient]
$v=-d \varepsilon_{\text {transverse }} / d \varepsilon_{\text {axial }} \equiv$ negative ratio of transverse to axial strain
$R e=\rho v d / \eta \equiv$ (inertial forces) $/($ viscous forces $)$
[ $d=$ characteristic length scale]
$W i \equiv($ characteristic time of fluid $) \times($ rate of deformation $)$
[e.g., Wi $=\tau \dot{\varepsilon}$ or $\tau \dot{\gamma}$ ]
${ }^{\text {a }}$ Sometimes the wall shear stress is specifically used in the denominator of the definition, or the ratio (viscosity $\times$ velocity)/length.
${ }^{\mathrm{b}}$ The definitions and uses of these groups are explained in detail in "Weissenberg and Deborah Numbers - Their definition and use", Rheol. Bulletin (The Society of Rheology), 79(2) p. 14 (2010).
${ }^{\mathrm{c}}$ Specific Deborah numbers are often used, depending on the choice of characteristic time. Popular examples are: Deborah number based on Rouse relaxation time ( $D e_{\mathrm{R}}$ ) and Deborah number based on longest relaxation time ( $D e_{\mathrm{d}}$ ).
${ }^{\mathrm{d}}$ The Poisson ratio is also defined as function of Young's and shear modulus ( $v=E / 2 G-1$ ) in three dimensions (and differently in two dimensions).
${ }^{\mathrm{e}}$ The Weissenberg number has sometimes been considered to be $N_{1} / \sigma$. However, this is a ratio of dependent rather than independent variables and thus describes data rather than experimental conditions. The quantity $N_{1} / \sigma$ is often called the stress ratio.
${ }^{\mathrm{f}}$ Specific Weissenberg numbers are often used, depending on the characteristic time. Popular examples are: Weissenberg number based on Rouse relaxation time ( $W i_{\mathrm{R}}$ ) and Weissenberg number based on longest relaxation time $\left(W i_{\mathrm{d}}\right)$.

## Table XIII. Frequently used constants

$N_{\mathrm{A}} \quad$ Avogadro's number, $6.023 \times 10^{23}$ molecules $\mathrm{mol}^{-1}$
$R \quad$ Ideal gas constant, $8.314 \mathrm{~J} \mathrm{~mol}^{-1} \mathrm{~K}^{-1}$
$k_{\mathrm{B}} \quad$ Boltzmann's constant, $1.38 \times 10^{-23} \mathrm{~J} \mathrm{~K}^{-1}$

