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TENTH ANNUAL MEETING

The tenth meeting of the Society was held, as announced in the November Leaflet, at the Mellon Institute of Industrial Research on December 28 and 29, 1938. The papers presented at the technical sessions provided a full program, and provoked much of the discussion which is one of the most valuable features of our meetings. While not formally planned as such by the Committee, the greater part of the program constituted a symposium on the relation of viscosity to the structure of liquids. The joint session with the Chemical Engineering Symposium on Fluid Dynamics was highly successful and served to bring the activities of the Society to the attention of a larger group of scientists interested in rheology.

Perhaps as a result of the Christmas season, the attendance at the meeting was somewhat below expectations, and we were particularly unfortunate in missing some of our members who have been most regular in their past attendance. Professor Bingham was recovering from a serious illness and was forced to send his greetings by wire. Professor Davey was somewhere in the South Pacific, Dr. Traxler was in Texas, and Dr. Hunter also was unable to be with us.

The local committee on arrangements had made excellent plans. Dr. E. W. Tillotson, as chairman, and also as official host for the Institute, made everyone feel very much welcome and quite at home. All of the guests were given the opportunity of a thorough inspection of the research facilities available, from the well equipped machine shop and semi-works equipment to the excellent as well as imposing library with its exquisite wood carvings and cheerful open fireplace. Most of the members took advantage of inspection trips through the Research Laboratories of the Aluminum Company of America, conducted by Dr. G. R. Sturm, or through the Westinghouse Laboratories, conducted by Dr. A. Nadai. Both of these companies have developments under way which illustrate interesting applications of the basic principles of rheology.

THE PROGRAM

With the exception of the omitted paper by Bingham and Adams on "The Fluidity of Electrolytic Solutions" the program was given as announced in Leaflet No. 7. Many of the papers gained interest from the models exhibited, the personalities of the speakers, and the discussions in which the group participated. Often it was necessary to close a lively discussion because of lack of time.

Molecular models used by M. L. Huggins to demonstrate the effect of kinking made more clear the possibilities of hydrogen bridge formation. The free rotation around C-C bonds in the rubber molecule was shown by models prepared by F. E. Dart and E. Guth in their interesting paper on the thermo-dynamics of stretched rubber.

Paul S. Roller brought out some of the great differences in the determinations of the plasticity of clay from similar work on solutions or homogeneous solids. The two-phase system of clay and water presents unique problems which have been attacked in a more suitable manner.

E. P. Irany presented a plea for more detailed study of the effect of inter-molecular forces upon the viscosity of solutions of polymer-homologous series and suggested methods for such a study. A paper which in some ways might be a companion piece was given by Arnold Kirkpatrick on the relations between chemical structure and plasticizing effect. The study of this field has been hindered by lack of apparatus for obtaining absolute units of flow. We shall probably find that the same laws hold in plastics as in fluids when the proper methods and apparatus are devised. For the last several years the program committee has attempted to find someone who would open up the discussion of plasticizers and we should congratulate Dr. Kirkpatrick for his courage in making the attempt. We hope that this will lead to more published information along this line.

The papers by Professors R. H. Ewell, R. B. Dow and Henry Eyring inaugurated a very profitable discussion about the fundamental nature and mechanisms of flow which we hope will be continued in the Rheology Leaflet.

COMMITTEE ON DEFINITIONS AND NOMENCLATURE

Although three quarters of an hour had been provided for discussion of the report which was published in the November issue of the Rheology Leaflet, program delays reduced the available time to about fifteen minutes. Drs. Mayo D. Hersey and A. Nadai spoke in opposition to the report as printed and no final action was taken on the report as a whole. The group did express their belief that the term "viscosity" should not be limited to simple liquids. Aside from the formal objection to this generalization of earlier scientific usage, the fact is that the word is widely used in industry to describe fluids in general and a limitation of its use by members of the Society would result in a failure to be understood. Wherever possible scientific terminology should be consistent with the vocabulary of the industrialist and technologist.

It is the hope of the officers of the Society that the entire question of definitions and nomenclature can be discussed in some detail in the coming numbers of the Leaflet and that at the next annual meeting, time may be allowed for a full and conclusive agreement.

THE RHEOLOGY LEAFLET

After the Annual Dinner on December 28, those present held an informal discussion of publication policy. This was done in order to obtain an expression of opinion from as many members as possible under conditions more favorable than those normally provided by the business meeting. This proved to be a very successful procedure and a thorough discussion disclosed a virtual unanimity of opinion on the following points:

1. That our present policy of submitting contributed articles, particularly those on original research, for publication in the Journal of Applied Physics should be continued.

2. That the Leaflet should be enlarged and improved by the inclusion of additional technical material.

The proposal that was most favorably received was for the inclusion of invitation review papers. It was agreed that abstracts and bibliographies should be limited to those covering specific topics and that we should not attempt any complete coverage of rheological literature. Reviews and abstracts will be given of certain books and of the more important articles. Discussions of the Definitions Report could very well be carried to such a point in the Leaflet that final decisions could be made during the annual meeting.

The discussion at the dinner meeting included consideration of the size of the membership in relation to publication of the Leaflet. The present membership is too small to support the Leaflet and its publication is in part paid for by reserves accumulated in previous years. It is essential that a substantial increase in membership be obtained in the next two years if the Leaflet is to be continued. It was therefore agreed at the meeting that all members should aid the officers in distributing complimentary copies of the Leaflet in an effort to increase the membership.

With this objective in mind, the editors have solicited papers which, we believe, will be of wide appeal to the several branches of rheology. It seems particularly fitting that for our first contribution Dr. Melvin Mooney, President of the Society, has consented to present a paper on the fundamental concepts of shear. This paper should be of great value to anyone concerned with deformation problems who is not familiar with the formal mathematical analysis of such problems, as it provides a summary of the fundamentals of this analysis in a relatively simple geometrical presentation.

STRESS AND RATE OF STRAIN
AN ELEMENTARY ANALYSIS

1. Introduction

When is the rate of shear not equal to the velocity gradient? What is the relationship between stress and rate of strain in three-dimensional, viscous deformation?

There are doubtless many practical rheologists whose knowledge of the theory of deformation of continuous media is adequate for most purposes, but not adequate to answer the above questions. The reason is that the texts which cover such questions completely and accurately are all highly mathematical in their presentation of the subject. However, a fairly complete analysis of homogeneous stress and strain can be made by means of simple geometrical diagrams and a little elementary algebra. Such an analysis will be presented in the present discussion.

The customary approach to the general problem of stresses due to viscosity is to assume the simplest possible relationship, that is, stresses proportional to corresponding rates of strain. Then it is shown that, for isotropic material, a certain proportionality factor introduced in the general, three-dimensional problem, can be identified as the coefficient of viscosity. This identification is made by applying the general formula to a two-dimensional simple shear. The method is powerful and entirely rigorous. Never the less it leaves the average rheologist completely mystified as to why, for example, the viscous tensional stress associated with a given rate of elongation, v_x, should have the value $2\eta v_x$. In the present discussion we shall start with the concept of viscosity in simple, two-dimensional shear; and we shall build up from that to the most general type of three-dimensional deformation.

Since the applications we have in mind are only to continuous viscous or plastic deformations not involving any elastic forces, we may limit the discussion to infinitessimal strains. Rates of strains, on the other hand, are considered finite.

2. Two-dimensional Strain

We consider a two-dimensional strain in the XY-plane. Suppose that the square section of the material, OABC of Figure 1, is transformed into the rhombus $O'A_1B'C_1$ of Figure 2, the sides of the rhombus being equal in length to the sides of the original square. For convenience we take this length as unity. If our reference axes have moved and rotated with the material, if necessary, in such a way that the X-axis remains coincident with the side OA, the axes take the positions $O'X_1$ and $O'Y_1$. The deformation with respect to these axes is described as a **simple shear** parallel to the X-axis. The **amount**

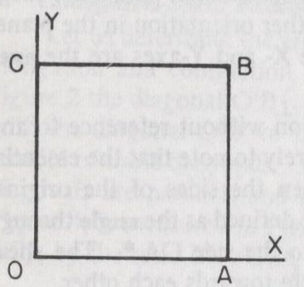

FIG. 1

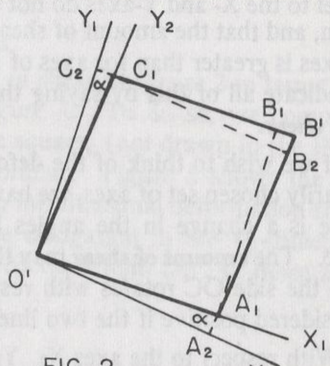

FIG. 2

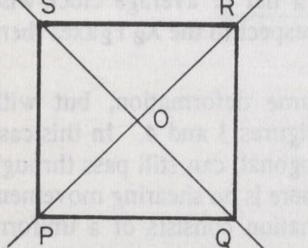

FIG. 3

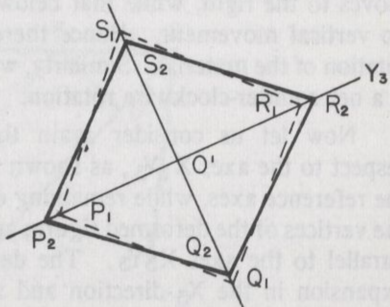

FIG. 4

of shear is the ratio $C_1C_2/O'C_2$, or, in the case of an infinitessimal shear, it is α, the angle of shear, expressed in radians.

Now suppose that our reference axes had moved and rotated during the deformation in such a way that the Y-axis remained coincident with the side OC. The axes would take the positions $O'X_2$ and $O'Y_2$; and the deformation would then be described as a simple shear parallel to the Y-axis. The amount of shear would still be the same, quantitatively, but would be measured by the ratio $A_1A_2/O'A_2$, or again the angle α.

We see therefore that the same shearing deformation can be equally well described as a simple shear parallel to the X-axis or parallel to the Y-axis. Furthermore, the shear parallel to the X-axis requires that there be an equal shear parallel to the Y-axis; and each of these two descriptions emphasizes only one of two aspects of the deformation, both of which are essential features of it. We may therefore appropriately refer to the deformation as an XY-shear. It is to be noted that this terminology implies not only that the shearing deformation takes place in the XY-plane, but also that the lines in the material parallel to the X- and Y-axes do not change in length during the deformation, and that the amount of shearing displacement parallel to these two axes is greater than for axes of any other orientation in the plane. We indicate all of this by saying that the X- and Y-axes are the **axes of the shear.**

If we wish to think of the deformation without reference to any arbitrarily chosen set of axes, we have merely to note that the essential change is a change in the angles between the sides of the original square. The **amount of shear** may thus be defined as the **angle** through which the side OC rotates with respect to the side OA.* The shear is considered positive if the two lines rotate towards each other.

With respect to the axes X_1 Y_1 the material above the X_1 -axis moves to the right, while that below moves to the left; and there is no vertical movement. Hence there is a net or average clock-wise rotation of the material. Similarly, with respect to the X_2Y_2 axes there is a net counter-clockwise rotation.

Now let us consider again the same deformation, but with respect to the axes X_3Y_3, as shown in Figures 3 and 4. In this case the reference axes, while remaining orthogonal, can still pass through the vertices of the deformed figure; and there is no shearing movement parallel to the axes X_3Y_3. The deformation consists of a uniform expansion in the X_3-direction and an equal uniform contraction in

*For a finite shear the amount of shear can be defined as the angle of rotation with respect to $O'X_1$, Figure 2, or a circle of radius OC, with center at O', and the circumference moving with material at the point C_2

the Y_3 -direction. It is easy to see that this is true if we think of the side Q_1R_1 as moving to the position Q_2 R_2 in two steps. If first the point Q_1 remains fixed and the point R_1 moves to R_2, the movement of any point on th eline is in the Y_3 -direction by an amount proportional to the Y_3 -coordinate of the point. In other words, it is a uniform expansion parallel to the Y_3-axis. Similarly, if R_2 now remains fixed and point Q_1 moves to point Q_2, the additional movement is a uniform X_3 contraction. There is shearing, of course, with respect to the bisectors of the angles formed by the reference axes.

The rotation of the material with respect to one of these bisectors is exactly balanced by an opposite rotation with respect to the other bisector. The net rotation with respect to either the bisectors or the X_3Y_3-axes is therefore zero. For this reason the deformation with respect to X_3Y_3 is described as a **pure shear.** If a shear is described as either simple or pure, with no mention of the reference axes, it is generally to be understood that the axes considered have a constant orientation in space. With this understanding, the shear in a capillary viscometer is a simple shear. A two-dimensional pure shear is seldom obtained in experiment.

3. Elongation-Shear Relationship

Let us determine the amount of shear in Figure 2 in terms of the elongation and contraction in Figure 4. To do so, we compare in Figure 2 the diagonal $O'B_1$ of the square, (not drawn in the Figure) with the diagonal O'B' of the rhombus. Figure 5 shows the upper termini of these diagonals. For an infinitessimal deformation the two diagonals are parallel; and e_y, the elongation in the Y_3 -direction in Figure 4, is measured in Figure 5 by the length DB', or more specifically, by the ratio $DB'/O'B_1$.

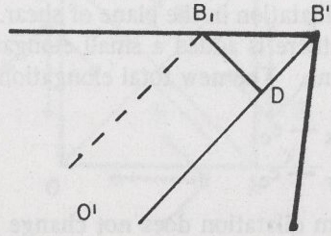

FIG. 5

The length of a side of the square or rhombus has been taken as unity. Then the following relationships are obvious:

(1) $O'B_1 = \sqrt{2}$,

(2) $B_1B' = \alpha$,

(3) $DB' = B_1B'/\sqrt{2} = \alpha/\sqrt{2}$,

(4) $e_y = DB'/OB_1 = \alpha/2$.

In like manner it is easily shown that the X_3 -elongation is

(5) $e_x = -\alpha/2$.

We thus see, incidentally, that

(6) $e_x + e_y = 0$.

If our original section of material is a unit cube, it is obvious that e_i is the increase in volume due to the elongation in the i-direction. Therefore equation (6) states the fact that the total volume change, or **dilitation,** is zero.

If the original section of material chosen in Figure 1 had been a circle instead of a square, the deformed section in Figures 2 and 4 would be an ellipsee, with its axes parallel to the X_3Y_3 -axes. It would be called the **strain ellipse,** and its axes the **strain axes.** The relations of Figures 2 and 4 tell us that in an infinitessimal shear the two diameters of the strain ellipse which retain their original length are orthogonal and at 45° with respect to the strain axes. With any other orientation of the reference axes the deformation would not appear as a shear only or as an elongation-contraction only, but as a combination of both types of strain.

We shall need later the relation between shear and the elongations when there is dilatation in the plane of shear. Suppose then that in Figures 2 and 4 there is added a small elongation e_o, equal in all directions in the plane. The new total elongations are then

$$E_x = e_x - e_o,$$

(7) $$E_y = e_y - e_o.$$

Since a uniform dilatation does not change angles the angle α and hence the shear remain unaltered. Then, from equations (4), (5) and (7), we find

(8) $$\alpha = E_y - E_x.$$

4. Rate of Shear

If the shear α, progressing at a constant rate, occurs in the time **t**, the rate of shear is α/t. We may say therefore that the **unit of rate of shear** is radians per sec. It is more customary to state this unit as cm per sec per cm. However, the former definition seems to be preferable in that it is more directly related to that essential feature of the deformation, the relative rotation of two particular lines in the material, and is not dependent upon the choice and proper movement of any reference axes. Hence in this discussion we shall denote rate of shear by ω, connoting angular velocity.

In this connection we point out that the rate of shear is not, in general, equal to the velocity gradient with respect to stationary axes, or axes of arbitrary orientation or rate of rotation. The Couette, or rotating cylinder, viscometer is an illustration of this point. We may state the following two equivalent correct definitions, applicable to a two-dimensional deformation of zero dilatation:

1. The rate of shear is the instantaneous velocity gradient normal to an axis which is and remains parallel to a line momentarily of constant length.

2. The rate of shear is the instantaneous relative angular velocity of two lines which are momentarily orthogonal and of constant length.

5. Two-dimensional Stress

Corresponding to the shearing deformations which we have been discussing, there is a stress system, proportional to the rate of shear, which we now wish to consider. In Figure 6 the vectors σ represent

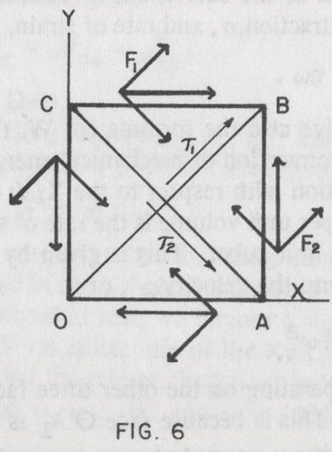

FIG. 6

the tangential stresses, or **tractions,*** which correspond to the shear and act on the four faces of the cube which are parallel to the Z-axis. Since the rates of shear parallel to the X_1 -axis and the Y_2 -axis are equal, as we have seen, the four tractions must be equal. Equality of these four tractions follows also from considerations of mechanical equilibrium, regardless of the origin of the stresses; but we can dispense with the additional proof.

We raise the question, What are the stresses operating on the planes through the diagonals OB and AC and parallel to the Z-axis? The answer is easily obtained by resolving the stresses σ in the two directions parallel to the diagonals. We see that there is a tension, τ_1 , across the face and negative tension or compression, τ_2 across the face OB. The magnitude of the tension τ_1 is given by the following series of equations. Assuming a unit cube,

(9) $$\tau_1 = (f_1 + f_2)/AC = \frac{2\sigma}{\sqrt{2}} / \sqrt{2} = \sigma .$$

Similarly,

(10) $$\tau_2 = -\sigma .$$

If there is a uniform tension or negative pressure, τ_o, in addition to the stresses already considered, we then have

(11) $$T_1 = \tau_1 + \tau_o ,$$
$$T_2 = \tau_2 + \tau_o ,$$

(12) $$\sigma = (T_1 - T_2)/2.$$

6. Stress-Rate of Strain Relationships—Two Dimensions

By the definition of the coefficient of viscosity, η, we have the relationship between traction, σ, and rate of strain, ω ,

(13) $$\sigma = \eta\omega .$$

We wish to derive also the formula for W, the rate of working per unit volume, or conversion of mechanical energy into heat. Considering the deformation with respect to the $X_1 Y_1$-axes, for example, in Figure 2, this rate per unit volume is the rate of work being done on the face C_1B' of the unit cube. This is given by the product of the shearing stress, σ, into the velocity ω , or

(14) $$W = \eta\omega^2 .$$

The tractions operating on the other three faces do not add anything to this work. This is because face $O'A_1$ is stationary; and the

*This use of the word traction is not standard usage. It is employed here to fill the obvious need for a single word to signify tangential stress, just as tension signifes normal stress.

faces $O'C_1$ and A_1B' only rotate about the Z-lines through O' and A_3, respectively, and hence have no component velocity in the direction of the tractions operating on those faces.

7. Stress-Rate of Strain Relationships—Three Dimensions

Just as any two-dimensional homogeneous deformation transforms a circle into an ellipse, so any three-dimensional homogeneous deformation transforms a sphere into an ellipsoid. The axes of the ellipsoid are called the **strain axes,** and the elongations along the strain axes are called the **principal elongations.**

If the deformation is initially described with reference to coordinate axes which are not parallel to the strain axes, the deformation involves, in general, both shears and elongations. The problem of determining the principal elongations and the orientation of the strain axes with respect to the reference axes in such a case is not a simple problem and will not be treated here. We can, however, analyze the stress-rate of strain relationships and determine the stresses acting on planes normal to the reference axes, regardless of whether or not the reference axes are parallel to the strain axes.

A. Shears Only.

Let us assume three shears with shear axes parallel to three orthogonal coordinate axes. It is obvious that these three shears and their corresponding three sets of shearing stresses can all exist simultaneously without effect upon each other. The tractions are given by three equations similar to (13),

(15) $\sigma_{yz} = \eta\, \omega_{yz}$,

and the total rate of working by

(16) $W = \eta\, (\omega^2_{yz} + \omega^2_{zx} + \omega^2_{xy})$.

B. Elongations Only.

Let the rates of elongation be v_x, v_y, v_z. Let the volume be constant. Then by the argument given in connection with equation (6),

(17) $v_x + v_y + v_z = 0.$

We shall make use of the fact that these elongations are equivalent to two shears, one in the XY-plane and the other in the YZ-plane. Starting with the material at rest, we impose a shear in the XY-plane with shear axes at 45° on either side of the X-axis. Let ω_z, the rate of shear, be $2v_x$, so that the rate of elongation in the X-direction is v_x and in the Y-direction is $-v_x$. By equation (13) the corresponding traction is

(18) $\sigma_z = \eta\omega_z = 2\eta\, v_x$.

We now propose to add a shear in the YZ-plane with axes lying at 45° on either side of the Z-axis. Before doing so we note that the shear already proceeding in the XY-plane involves a certain amount of shearing and dilatation in the YZ-plane. Although there is no movement parallel to the Z-axis, there is an elongation $-v_x$ along the Y-axis; and this means a uniform dilatation $-v_x/2$ in the YZ-plane, plus Y-and Z-elongations $-v_x/2$ and $v_x/2$, respectively. By analogy with equation (8), the rate of shear is v_x.

We now add the new shear, ω_x, with the same axes. Then the new shear is added algebraically to the YZ-shear already present; and the corresponding additional stress system is added algebraically to the first one. Let the added rate of shear be $2v_z$, so that the rate of elongation in the Z-direction is v_z and the added rate of elongation in the Y-direction is $-v_z$. The corresponding added traction is

(19) $\sigma_x = \eta\, \omega_x = 2\, \eta\, v_z$.

The two shears ω_z and ω_x which we have now imposed on the system produce a total rate of elongation in the Y-direction of $-v_x - v_z$. But by the equation (17) this is v_y. Hence all three rates of principal elongations now have their required values; and the two shears ω_z and ω_x together are equivalent to the three elongations v_x, v_y, v_z.

The tensions along the coordinate axes arising from these shears are, after equation (9),

$$\tau_x = 2\,\eta\, v_x\,,$$

(20) $$\tau_y = -\,2\,\eta\,(v_x + v_z) = 2\,\eta\, v_y\,,$$

$$\tau_z = 2\,\eta\, v_z.$$

The rate of energy dissipation is

(21) $$W = \tau_x\, v_x + \tau_y\, v_y + \tau_z\, v_z\,.$$

Substituting for τ_x, etc from (20) and applying (17), we obtain

(22) $$W = 2\eta\,(v_x^2 + v_y^2 + v_z^2).$$

C. Combined Shears and Elongations

If the deformations treated in the two previous sections proceed simultaneously, the displacements are added vectorially, as are likewise the stresses. Thus the resultant stress on a plane normal to the X-axis has the X, Y, Z,-components,

$$\tau_x = 2\,\eta\, v_x\,,$$

(23) $$\sigma_{xy} = \eta\, \omega_{xy}\,,$$

$$\sigma_{zx} = \eta\, \omega_{zx}\,.$$

The displacement of a surface normal to an axis, resulting from one of the deformations, has no component in the direction of the stress on that surface resulting from the other deformation. Hence the rates of working resulting from the two deformations are mutually independent and are algebraically additive. Thus,

$$W = \eta \left(\omega_{yz}^2 + \omega_{zx}^2 + \omega_{xy}^2 + 2 \left(v_x^2 + v_y^2 + v_z^2 \right) \right). \tag{24}$$

If there is an added uniform tension or negative pressure, τ_o, so that the total tensions are

$$\tau_x = \tau_x + \tau_o, \text{ etc.,} \tag{25}$$

then instead of (23) we have

$$\begin{aligned} \tau_x - \tau_o &= 2\,\eta\, v_x , \\ \sigma_{xy} &= \eta\, \omega_{xy} , \\ \sigma_{zx} &= \eta\, \omega_{zx} , \text{ etc.} \end{aligned} \tag{26}$$

From equations (20) and (17) it is obvious that

$$\tau_x + \tau_y + \tau_z = 0. \tag{27}$$

Then by adding together the three equations of type (25), we see that

$$\tau_o = \frac{1}{3} (T_x + T_y + T_z). \tag{28}$$

That is, τ_o is the mean tension.

We can illustrate the use of equations (26) and (28) by considering the measurement of viscosity by the method of stretching a glass thread under constant tension. Let τ_3 be the tension, applied in the Z-direction. If the atmospheric pressure is p, then

$$\tau_o = - p + \tau_3/3 , \tag{29}$$

$$\begin{aligned} \tau_x &= - p \\ \tau_y &= - p \\ \tau_z &= - p + \tau_3 \end{aligned} \tag{30}$$

$$v_x = v_y = - \frac{1}{2} v_z . \tag{31}$$

Substitution of (29), (30.1) and (31) into (26.1) yields

$$\eta = \tau_3/3\, v_z \tag{32}$$

8. Some Theorems Concerning Three-Dimensional Deformations

Suppose we have two shears, α and β with one common axis, OY, as shown in Figure 7. Then the resultant displacement of any point at the height y above the VX-plane is given by the vector sum $y\ (\alpha \mathbf{i} + \beta \mathbf{j})$, where **i** and **j** are unit vectors in the directions of the two shear axes OX and OV, respectively. The deformation is obviously a single shear in the YW-plane. Hence we have

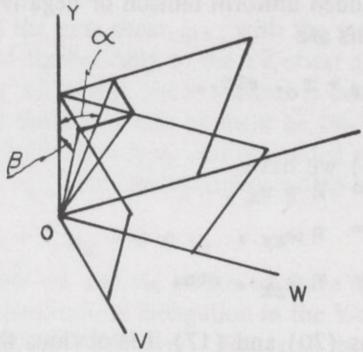

FIG. 7

Theorem I. Any two shears having one shear axis in common are equivalent to a single shear with one axis the same as the common axis. The other axis and the value of the resultant shear are determined by the vector sum of the two displacements equal respectively to the two original shears and directed parallel to their two axes which are not common.

It has been stated that any homogeneous strain is equivalent to three elongations along properly oriented orthogonal axes. As a rule problems in elasticity and plasticity are discussed with reference to these principal elongations. However, a rheologist who happens to deal mostly with liquids and the problems of viscous flow may have difficulty in thinking in terms of elongations. It is natural to inquire whether a homogeneous strain in general is equivalent to three shears with properly oriented orthogonal shear axes.

In order to answer this question let us consider a sphere which we deform infinitessimally at constant volume into an ellipsoid. The sum of the principal extensions must be zero, or

(33) $$e_x + e_y + e_z = 0.$$

Now let us assume the surface of the ellipsoid and that of the original sphere superposed with their centers coinciding. Since by equation (33) all three elongations cannot have the same sign, the two surfaces will cross along certain lines, which we shall call the neutral lines. There are two such lines, each forming a complete loop around that axis along which the elongation is opposite in sign to the other two elongations. The loops are symmetrically situated on opposite sides of the sphere.

A radius of the sphere to any point in either of the neutral lines remains of constant length during the deformation. Let such a radius determine a new X'-axis. Then

(34) $$e_{x'} = 0.$$

Now we pass a plane through the center of the sphere and normal to the X'-axis. This plane cuts the ellipsoid in an ellipse having its center at the common center of the sphere and ellipsoid. Let any two orthogonal radii of the ellipse be drawn, and denote the corresponding elongations by $e_{y'}$ and $e_{z'}$.

If we imagine a unit cube with its faces normal to these three radii, or axes, we see that the total dilatation associated with the corresponding elongations is the sum of the elongations. The cube is subjected also to shears parallel to its faces; but we have seen in Section 3 that there is no dilatation resulting from shears. Hence, since the dilatation is zero, we must have

(35) $$e_{x'} + e_{y'} + e_{z'} = 0.$$

Then by equation (34),

(36) $$e_{y'} + e_{z'} = 0.$$

Therefore there is no dilatation in the plane of the ellipse. This being the case, there are two orthogonal radii of the ellipse which retain their length during the deformation. They are therefore radii of the original sphere and intersect the surface of the sphere at two other points on the neutral lines.

We have thus found three mutually orthogonal radii of the sphere which remain of constant length during the deformation. The deformation with respect to these three radii as reference axes can only be three shears. Since the first radius was drawn to any point on the neutral line, there is an infinite number of such sets of orthogonal axes which resolve the deformation into shears without elongations along the axes. We thus have the

Theorem II. Any homogeneous infinitessimal deformation of zero dilatation is equivalent to three shears with respect to three orthogonal axes, properly oriented with respect to the strain axes of the deformation.

If the X'-axis is directed through a certain point, some interesting simplifications result. Suppose that e_z is of one sign and e_x and e_y of the opposite sign. Then the neutral lines are loops around the Z-axis. Let the X'-axis be directed through the point where the neutral line intersects the YZ-plane in the quadrant -Y and +Z, as shown in Figure 8. The Y'-and Z'-axes are symmetrically directed in the two octants +X, +Y, +Z and -X, +Y, +Z. The X', Y', Z'-axes thus enclose the Z-axis.

The precise directions of the new axes with respect to the strain axes, and the shears in the planes of the new axes, can be expressed in terms of the principal elongations. The calculations involve only algebra, but they are too lengthy for presentation here. The results are as follows:

Direction Cosines With Respect to the Strain Axes

	X'	Y'	Z'
l	0	$1/\sqrt{2}$	$-1/\sqrt{2}$
m	$\sqrt{\dfrac{e_z}{e_z-e_y}}$	$\sqrt{\dfrac{e_y}{2(e_y-e_z)}}$	$\sqrt{\dfrac{e_y}{2(e_y-e_z)}}$
n	$\sqrt{\dfrac{e_y}{e_y-e_z}}$	$\sqrt{\dfrac{e_z}{2(e_z-e_y)}}$	$\sqrt{\dfrac{e_z}{2(e_z-e_y)}}$

The shears are

$$a_{y'z'} = \sqrt{-2\, e_y\, e_z}$$

$$a_{z'x'} = \sqrt{-2\, e_y\, e_z}$$

$$a_{x'y'} = -2\, e_x .$$

As could be shown entirely from symmetry considerations, we see that $a_{y'z'} = a_{zx}$ Hence if these two shears are combined in accordance with Theorem I, the resultant shear, $a_{x'}$, will have one axis collinear with the X'-axis and the other axis lying in the Y'Z'-plane and bisecting the angle between the + Y' and +Z' axes. The value of the shear is $\sqrt{a^2_{y'z'} + a^2_{z'x'}}$ or

$$a_{x'} = 2\sqrt{-e_y\, e_z} .$$

In view of the fact that any deformation can be expressed as three elongations along properly oriented orthogonal axes, we have now established

Theorem III. Any infinitessimal deformation of zero dilatation is equivalent to two shears lying in two orthogonal planes, one shear having one axis at 90° and the other at 45° with respect to the axes of the other shear.

Experimental measurements of viscosity in three-dimensional shear would be of great interest, especially as applied to thixotropic materials or to colloidal solutions which exhibit streaming anisotropy. In the latter case, at least, we could anticipate a different viscosity in different planes of shear. However, it seems inherently impossible to devise an apparatus for maintaining continuously a three-dimensional shear that is homogeneous throughout any large fraction of the material under test. Apparently, then, any possible experiment will involve the difficulties of integrating some very complex field equations with a set of complex boundary conditions.

Figure 8

The case illustrated is with the principle elongations

$$e_x = \delta$$

$$e_y = 2\delta \qquad |\delta| \ll 1$$

$$e_z = -3\delta$$

ADCX' is a neutral line, or line of intersection of the original spherical surface with that of the ellipsoid produced by the strain. The other neutral line is the mirror image in the XY-plane of the one shown. Regardless of the ratios of the elongations, the neutral lines always pass through the normals to the sides of the regular tetrahedron determined by the XYZ-axes. Four of these points are shown at E, E', F and F'. The line ABC, a section of an ellipse, is the projection of the neutral line in the ZX-plane.

X' is the intersection of the neutral line with the YZ-plane. Y' and Z' are mirror images of each other in the YZ-plane and form with X' an orthogonal system of axes. With respect to these axes the deformation consists of shears only. The X'Y' and Z'X' shears are equal.

A similar symmetrical solution would be obtained if one of the new axes were passed through any of the points A, D, or C.

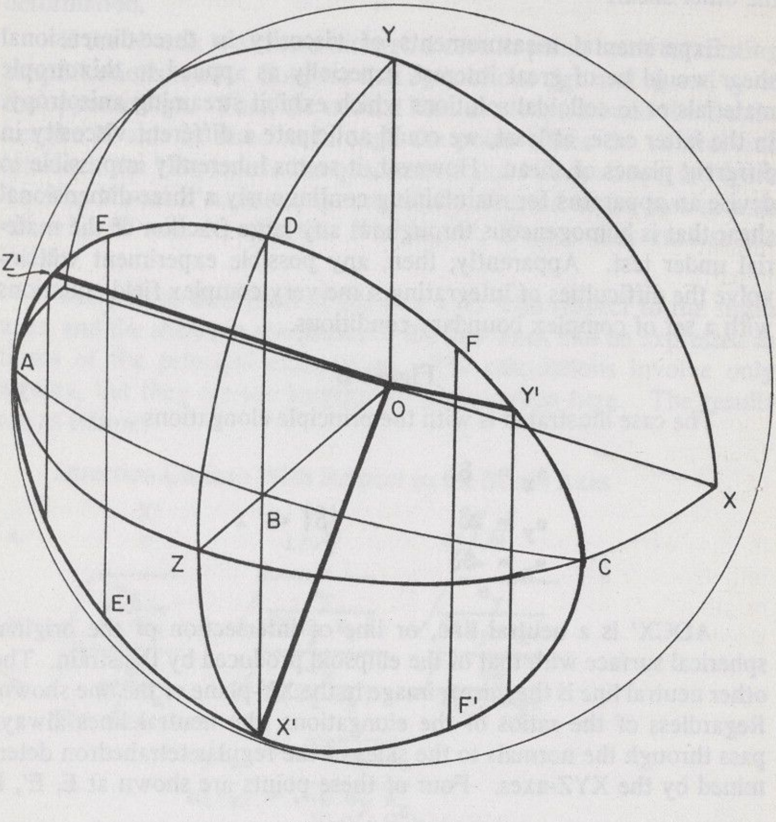

FIG. 8

G. W. Scott Blair, An Introduction to Industrial Rheology.
(Blakiston, Philadelphia, 1938) 143 pp.

The volume does not pretend to be a textbook on a definitely outlined field of science but rather, a loose collection of treatises on a science in the making. The author attempts a systematic review of what has been done in the field which, in due course, will be consolidated into a science of industrial rheology.

In order to appreciate the difficulty of the task, one must consider the relative immature state of rheology as an exact science—the controversial nature of its most fundamental conceptions, the confusing growth of terms and definitions, the frequently meaningless empiricism of mathematical expression, the mass of minutiae waiting for significant correlation and generalization.

An introduction to industrial rheology should be suggestive of a mission of this science; it should not only gather the widely scattered material but organize, digest, discriminate. It may be questioned whether in this sense, the author has offered more than an uncritical review. Much of the material dealt with at great length is too insignificant and confusing to encumber an introduction to a new science; in particular, too much prominence is given to rather meaningless algebraic formulations which do not present rheology in its best light to those who, for some practical reasons, desire an introduction to it.

The author, who has notably contributed to the present stock of knowledge, has well enough succeeded in presenting—if not precisely an introduction to industrial rheology in bold outline—a good survey of its fragmentary material. The presentation of fundamentals is brief, concise, and up-to-date. Considering the lack of general summaries in this field, the book should be interesting to the novice as well as the initiated—if, indeed, anyone may find himself altogether ignorant of, or all-inclusively expert on so wide and ramified a subject. A foreword by Professor E. Bingham and a glossary of rheological terms add to the value of the volume which closes with the plea that physicists, chemists, engineers and technicians in all branches of industry join forces as rheologists, conscious of a common task. E. P. Irany.

BIBLIOGRAPHY

The Viscosity of Solutions and Pure Liquids

The Flow of Substances with Flow Resistance.
W. Philippoff. Kolloid-Z. 83, 163-72 (1938).

Viscosity of Fluids and Molecular Volume—Principle of Continuity of State

D. T. Lewis. J. Chem. Soc. 1061-66. July, 1938.

A Correlation of Intermolecular Forces and Fluidity and Density Data

J. Newton Friend. Trans. Faraday Soc. 34, 813-16 (1938).

Deviatiton of Viscosimetric from Mean Molecular Weight

G. V. Schulz. Z. physik. Chem. 41, Abt. B. 6, 466-68 (1938).

THE SOCIETY OF RHEOLOGY

The Society of Rheology was founded in 1929 to further the study of the deformation and flow of matter. This purpose has been interpreted in the broadest sense, as covering types of deformation ranging from the viscous flow of fluids, through the plastic flow of soft substances, to the elastic deformation of solids. Rheology may be regarded as the science whose industrial application constitutes the field of materials testing.

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