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August, 1939

PROGRAM

ELEVENTH ANNUAL MEETING NATIONAL BUREAU OF STANDARDS OCTOBER 13-14, 1939

SOCIETY HEADQUARTERS:

Wardman Park Hotel

Connecticut Ave. and Woodley Road, N. W., Washington, D. C.
Rates \$3.00 single; \$5.00 double.

The hotel will arrange sightseeing tours for the families of members.

The Society is deeply in debt to the local committee on Arrangements. Chairman J. W. Burney is being assisted by W. H. Herschel, A. T. McPherson and Paul S. Roller.

Ten years ago the Society held a very successful meeting at the Bureau. During these intervening years the Bureau has been the center of a large amount of work along rheological lines. The committee has made it possible for our members to gain an intimate view of these activities which should be both interesting and of considerable practical value.

The Program Committee consisting of R. H. Ewell, chairman, E. C. Bingham, J. H. Dillon, P. D. Foote, W. H. Herschel, A. Nadai, E. W. Tillotson and R. N. Traxler, has cooperated with the local committee and has arranged a short but important series of papers for the Saturday morning session.

October 13th

9:00-9:45 A. M. Registration. East Building. National Bureau of Standards.

The registration desk will endeavor to function as an agency for making contacts between visitors and Washington scientists.

10:00 A. M. Symposium on Rheology in Science and Industry. Lecture Hall. East Building.

Greeting: Welcome by Dr. Lyman J. Briggs, Director of the National Bureau of Standards.

1. Introductory Remarks. A. S. Hunter, E. I. du Pont de Nemours and Co.
2. Rubber. J. H. Dillon, Firestone Tire and Rubber Co.
3. Paint and Varnish. G. G. Sward, Institute of Paint and Varnish Research.
4. Glass. G. W. Morey, Geophysical Laboratory.
5. Geophysics. L. H. Adams, Director, Geophysical Laboratory.
6. Soil Mechanics. C. A. Hogentogler, Bureau of Public Roads.
7. Clay Products and Masonry Mortar. J. W. McBurney, National Bureau of Standards.
8. Plastics and Organic Fibres. W. E. Emley, National Bureau of Standards.
9. Dental Materials. Wilmer Souder, National Bureau of Standards.
10. Concrete. W. H. Herschel, National Bureau of Standards.
11. Medicine and Biology. Dr. F. S. Brackett, National Institute of Health.
12. Metals. A. Nadai, Westinghouse Electric and Manufacturing Co.

1:00 P. M. Luncheon at the Cafeteria, Industrial Building, 75 cents.

2:00 P. M. Inspection trips starting from in front of the Industrial Building.

There are a number of investigations of rheological interest in progress at the Bureau of Standards and elsewhere in Washington which are not represented in the Symposium. Arrangements have been made for consideration of these topics in connection with the inspection trips. Among such items are work on the absolute viscosity of water, an investigation of high pressure lubrication, viscosity of oil and various aerodynamic studies. Although a general tour of the Bureau has been arranged, members of the Society and guests can spend the afternoon in the section or laboratory in which they are particularly interested.

The following inspections trips have been provided:

1. National Bureau of Standards.
2. Geophysical Laboratory of the Carnegie Institute.

3. Laboratories of the Bureau of Public Roads.
4. Institute of Paint and Varnish Research.
5. Technical Laboratory of the Federal Bureau of Investigation.

Arrangements can be made for small groups to visit any of a number of other governmental and private laboratories in Washington and vicinity.

October 14th

9:30 A. M. Lecture Hall. East Building.

Report of the Committee on Definitions and Nomenclature.

10:30 A. M. M. Mooney. U. S. Rubber Co., Passaic, N. J. "A Three-Dimensional Model to demonstrate the relationship between Principal Strains and Simple Shears."

10:40 A. M. Raymond B. Block. Wright Field, Dayton, Ohio. Note on the use of The Rolling-Ball Viscometer for Measuring the Effect of Pressure upon the Viscosity of Liquids."

11:00 A. M. R. B. Dow. The Pennsylvania State College, State College, Pa. "Computation of some Physical Properties of Lubricating Oils at High Pressures."

11:30 A. M. E. C. Bingham. Lafayette College, Easton, Pa. "Neglected Fields of Rheology." This paper will cover reaction velocities, the role of rheology in the bodies of animals and plants in sickness and health, the physical property of hardness from a rheological point of view and the rheological importance of "film."

12:00 G. W. Scott Blair and F. M. V. Coppen. National Institute for Research in Dairying. University of Reading, Reading, England. "Psycho-Rheology."

1:00 P. M. Luncheon. Wardman Park Hotel, price \$1.00.

2:00 P. M. Business Meeting. Wardman Park Hotel. Members interested in the future of the society should arrange their plans so that they may attend this meeting.

At the Tenth Annual Meeting held December 28 and 29th in Pittsburgh, two papers were presented which seem to deserve further consideration and are therefore printed in this issue. In the absence of Professor Gemant his paper was read by H. R. Lillie. It represents an able attempt at analysis of a very complex and controversial subject and it is presented in the hope that it will stimulate further research on the electrical properties of plastics.

The paper of M. Reiner and K. Weisenberg was received after the meeting was over and is given at this time so that it may be discussed at the coming meeting.

THE ELECTRICAL CONDUCTIVITY OF PLASTIC DIELECTRICS

By Andrew Gemant

Department of Electrical Engineering, University of Wisconsin,
Madison, Wisconsin

* * *

The author's theory on solid friction is here applied to ions in a plastic dielectric, and it is shown how the experimentally observed behavior of the electrical conductivity under varying field strength on the one hand, and under varying frequency of the impressed voltage on the other can be explained on the above basis.

* * *

In liquids, ionic mobility is controlled by the viscosity of the medium, and the viscosity of the medium with respect to motion of the ions is that obtained from large scale measurements. There is no doubt that this rule is valid for liquids of very high viscosity as well. In dealing with elasto-viscous liquids, however, the rule is likely to fail, its failure becoming quite evident, if plastic solids are being considered. Two alterations have to be introduced into the original rule valid for liquids. **First**, there is no a priori justification to suppose that the viscosity the ions have to overcome should have the same numerical value as that obtained from large scale measurements. **Second**, it is certain that the elastic properties of the solid will have to be taken into consideration, just as for any other deformation of the plastic. The first alteration mentioned is certainly important, but unfortunately scarcely anything definite is known on this point. This paper will therefore be concerned chiefly with the second alteration proposed.

The author has published some work on the rotation of dipolar molecules (1) in vitreous solids, and has mentioned in those papers that it is possible to extend those ideas to the motion of ions as well. The fundamental conception is that generally used in connection with deformation of plastics, (2) namely, that each deformation is a super-

1) A. Gemant: Jour. Appl. Phys., 9, 730, 1938.

2) J. M. Burgers: First Report on Viscosity a. Plasticity, Amsterdam, 1935.

position of an elastic displacement and a viscous flow. The displacement may be different for both of these components and will depend upon special experimental conditions, but since the stress for both the elastic displacement and the viscous flow is the same, they can be considered to be mechanically connected in series. The assumed elastic displacement is not ideal, but is coupled with a loss mechanism, known as solid friction. Being an integral part of the elastic displacement, as known from experiments on mechanical vibrations below the yield point, the solid friction must be considered as connected mechanically parallel to the elastic element.

The next step is to specify these three fundamental units of resistance, assumed to oppose the motion of ions, in accordance with existing data.

It has just been stated that no experimental method for determination of the **viscous component** exists up to now. The electrical conductivity of the material in question, determined by means of d.c. and at low field strength, allows the ratio of the ionic concentration to the viscosity to be computed. In other words each value assumed for the viscosity will fix the value of the concentration of ions as well. We then have to choose a coupled pair of these values such that both should appear as much in accord with experimental evidence as possible. In this way we can estimate in a two-fold manner the viscosity, thus restricting the plausible range considerably. How this has to be carried out will be shown below.

As to the **elastic** component, we assume an elastic modulus for the surroundings of the ion, equal to that known from large scale experiments, although again it is possible that the true value differs somewhat from the assumed one. The **solid friction** component also can be estimated from numerical (3) data available on several vitreous materials with ionic conductivity, (glass, ebonite, polystyrene, etc.). The most striking feature of these data is that the frictional term, expressed as logarithmic decrement of vibrations, or as specific loss per cycle, is fairly independent of the frequency from the lowest up to around 10^5 cycles per second. We can therefore hardly speak of a dissipative term, like the viscous resistance, but rather of a resistance the value of which varies inversely with the impressed frequency.

The final equations depend on whether the impressed voltage is direct or alternating. The details of the above are being published elsewhere. The purpose of the present paper is to give some numerical computations based on our equations, and to compare the result with some experimental data of the literature. Let us first consider the case of a **d.c. voltage**. The application of our combined resistance

3) A. L. Kimball: *Vibration Prevention in Engineering*, New York 1932, p. 133.
A. Gemant a. W. Jockson: *Phil. Mag.* 23, 960, 1937.

shows that for small field strengths the viscous component comes into action. The elastic displacements of the ion are, however, even for the highest fields applicable, small as compared with the viscous ones. In consequence of this the solid friction element will have practically no importance in this connection. The increase of the ionic mobility, due to elastic shifts, also appears to be negligibly small. However, a rather plausible assumption seems to be that the amounts of kinetic energy collected by the ion during its transitions from elastic to viscous displacements will, to a certain extent at least, accumulate as vibratory energy. The ionization number α , number of successful impacts during a path of 1 cm., can be shown to be:

$$\alpha = \frac{e^2 E^2}{L_1 \mu \lambda^2} \quad (1)$$

where e is the charge of a monovalent ion, E , applied electric field, L_1 the ionization energy, μ the shear modulus of elasticity, and λ the molecular diameter with a numerical factor left open. Eq. 1 gives maximum figures, the true ones are probably smaller. For numerical calculations let us assume a material with the following constants, in absolute units: $L_1 = 10^{-11}$ (corresponding to an ionization potential of 6 volts), $\mu = 10^{10}$, $\lambda = 7 \times 10^{-8}$ cm. Then if E is in absolute units,

$$\alpha = 5 \times 10^{-4} E^2 \quad (2)$$

The current-voltage characteristic of this material should now be calculated.

If two electrodes of 1 cm^2 surface are placed at a distance of a cm., and the original concentration of the ions is n , then, in case internal ionization occurs, the final concentration n_E becomes, (4) for $\alpha a > 1$

$$n_E = \frac{n}{\log 2\alpha a} (2^{\alpha a} - 1) \quad (3)$$

and the ratio of conductivities σ for a field E and for small fields:

$$\frac{\sigma_E}{\sigma_0} = \frac{1.4}{\alpha a} (2^{\alpha a} - 1) \quad (4)$$

Assuming $\sigma_0 = 2.5 \times 10^{-3}$ (about 3×10^{-15} mhos per cm.) we have for the current density at field E:

$$i = \frac{3.5 \times 10^{-3} E}{\alpha a} (2^{\alpha a} - 1) \quad (5)$$

and, substituting Eq. 2, with $a = 0.05$ cm.,

$$i = \frac{140}{E} (22.5 \times 10^{-5} E^2 - 1) \quad (6)$$

Evaluating Eq. 6 up to 150 kV/cm., Fig. 1 is obtained.

The literature contains quite a number of such current-voltage characteristics, showing a similar behavior to Figure 1. We quote here as an example data of W. Scislowski on paraffin wax, (5) and those of S. Whitehead on cellulose acetate and varnish paper board, (6) solids with finite plasticity.

Table 1 contains our figures together with those calculated from curves given by the two authors just quoted; it can be seen that our assumed data lie between those referring to paraffin wax, a high grade insulator, and paper board, a rather poor one.

Table 1.
Current density dependent on field strength

Data on paraffin wax after Scislowski		Data on varnish paper board after S. Whitehead		Data calculated from Eq. (6).	
Kv/cm	10^{-12} Amp.	Kv/cm	10^{-6} Amp.	Kv/cm	10^{-9} Amp.
25	0.0005	60	3.3	35	0.1
50	0.0035	120	7.5	70	0.32
75	0.014	180	17.	105	0.97
100	0.04	240	40.	140	2.8

The essential features of the behavior of the conductivity are as follows. Deviation from Ohm's Law generally starts between 20 and 100 kV/cm., according to the thickness of the sample, the initial field strength increasing with decreasing thickness. Near the initial field strength our Eq. 4, in conjunction with Eq. 2, indicates that the conductivity follows an equation of the kind:

$$\sigma_E = \sigma_0 + \beta E^2 + \gamma E^4$$

4) N. Semenoff and A. Walther: Westigkeitslehre, Berlin, 1928 page 150.

5) W. Scislowski: Acta Phys. Polonica, 4, 123, 1935.

6) S. Whitehead: World Power, 26, 72, 1936.

in which β and γ are constants, as found by L. Hartshorn for varnished cloth. (7) For higher fields the increase of current is of an exponential nature. For fields between 200 and 1000 kV/cm. (again increasing with decreasing thickness) the current density becomes the order of several microamperes, and instability generally follows.

We now turn to the effect on ionic conductivity of the frequency of the impressed voltage. For **a.c. voltages** the conductivity is separated into a real and imaginary term, the latter standing for the dielectric constant. The ratio of the two terms is then the loss angle, as usually measured. In the following, calculations of the loss angle will be carried out.

The theory as outlined above indicates that the ionic mobility, controlled by the viscosity for d.c., will be determined by the viscosity, elasticity, and solid friction together, when the frequency increases. Above a certain frequency range, which is lower the larger the viscosity, the resistance will be determined by the elasticity and solid friction alone. For temperatures considerably below the softening point the viscosity constant of solids is so large that the frequency range just mentioned will be below 1 cycle per second. This means that for the whole practical frequency range the loss angle is only a function of the elasticity and solid friction.

We are further restricted to frequencies below say 10^5 cycles, as we do not possess any data on the solid friction coefficients for higher frequencies. In drawing comparisons with experimental data one has to bear in mind that apart from ionic motion there are other specific mechanisms leading to dielectric losses. Our formulae will, therefore, only supply a certain fraction of the total loss, which fraction will be the higher, the less pronounced these specific mechanisms are in any special case. The other mechanisms just referred to are due to inhomogeneities of the material (Maxwell-Wagner), and to dipolar molecules (Debye).

The final result of the corresponding deductions is that for sufficiently low temperatures the loss angle, $\tan \delta$, is:

$$\tan \delta = \frac{2ne^2\theta}{\lambda \epsilon \mu} \quad (7)$$

where n = ionic concentration, θ = specific loss of solid friction, and ϵ = dielectric constant.

Let us evaluate Equation 7 for soda glass at different frequencies. Take $\lambda = 7 \times 10^{-8}$, $\epsilon = 5$, $\mu = 2.5 \times 10^{11}$. In glass, especially

7) S. Whitehead: Proc. Phys. Soc. 47, 974, 1935.

soda glass, it is safe to assume that the order of magnitude of n is the same as the order of magnitude of the number of molecules, n_m , namely 10^{22} . This estimate can be strengthened somewhat in connecting it with an estimate of η_0 , the ionic viscosity. We have for the d.c. conductivity at small fields the general formula:

$$\sigma_0 = K \frac{n}{\eta_0} \quad (8)$$

where K is to a first approximation independent of the material chosen since the molecular radii which enter Stokes' equation are always of the same order. K can be estimated from oils to be about 10^{-12} abs. Thus for glass with $\sigma_0 \sim 10^{-3}$

$$\frac{n}{\eta_0} \sim 10^9$$

With $n = 10^{22}$, η_0 becomes 10^{13} . The extrapolated value from large scale measurements is 10^{15} for η_0 . In a recent paper on the slow changes in physical properties of glasses E. Seddon (8) also comes to the conclusion that η_0 must be high but finite. In assuming lesser orders for n , η_0 had to be decreased proportionally which, however, seems unlikely, considering the extrapolated value. Thus the pair $n \sim 10^{22}$, $\eta_0 \sim 10^{13}$ seems the most probable.

It has already been stated that θ is fairly independent of frequency. This independence also becomes evident by a recent paper by Kruger and Rohloff on the internal friction of wood (9) in a frequency range of from 10 to 10^4 cycles per second. Using then for glass the data as obtained by Gemant and Jackson, we have the curve A of Figure 2. Using, on the other hand, data obtained by Bennewitz and Rotger, (10) a slight variation with the frequency is noticeable (curve B of Fig. 2), with a flat maximum of around 0.7 cycles. Figure 2C is obtained from data of Wegel and Walther on glass (11) at a frequency range 10^3 to 10^5 . It appears that there is a slight decrease of θ , and, according to Eq. 7, of the loss angle when the frequency increases.

These conclusions may be compared with the data by M. J. O. Strutt. (12) The loss angle of glasses for any given temperature varies but slightly with frequency according to his data, becoming generally less with increasing frequency. This is the same result as seen on

- 8) E. Seddon: J. Soc. Glass Technol. 22, 268, 1938.
 9) F. Kruger and E. Rohloff: Zeits. f. Phys. 110, 58, 1938.
 10) K. Bennewitz a. H. Rotger: Physik., Zeits. 37, 578, 1936.
 11) R. L. Wegel a. H. Walther: Physics, 6, 141, 1935.
 12) M. J. O. Strutt: Arch. f. Elektrotechnik, 25, 715, 1931.

Figure 2. The absolute value of $\tan \delta$ is between 10^{-3} and 10^{-2} for room temperature, somewhat higher than the values indicated in Figure 2. Curve D in Fig. 2 refers to one of his sets on a soda glass.

Although Eq. (7) is not valid for elevated temperatures, a temperature curve of $\tan \delta$ for lead glass, based on solid friction data of Gemant and Jackson, and on Eq. (7) is shown in curve A of Fig. 3. Until the ionic viscosity, η_0 , is more definitely known, the use of the more general equations, as supplied by the theory, would offer but little advantage. Curve B is based on direct measurements by Strutt of $\tan \delta$ of a lead glass. The increase with temperature of $\tan \delta$ is, as can be seen, of an exponential nature, the factor in the exponent being usually smaller than the corresponding factor for the d.c. conductivity, the a.c. and d.c. curves submerging asymptotically towards higher temperatures.

SUMMARY

1. The paper is concerned with ionic mobility in vitreous dielectrics, which generally exhibit a finite plasticity. The fundamental idea is to deal with the deformation as caused by the motion of an ion in the same way as macroscopic deformations of plastics are dealt with. This latter can be described by means of three constants of the material, namely, plasticity, elasticity, and solid friction, and the same should hold for the molecular deformation around a moving ion. The equations for the ionic mobility, as deduced by the author in another paper, are given here for numerical computation and comparison with some experimental data.

2. In the case of d.c. the current-voltage characteristic can be calculated. This is done for a set of assumed constants of a plastic dielectric up to a field of 150 kV/cm. The curve shows the same features as measurements on paraffin by Scislowski, and on cellulose acetate and varnish paper board by S. Whitehead.

3. For an a.c. voltage the dielectric loss angle can be calculated from the theory. This is done for soda glass, based on solid friction data of Gemant and Jackson, Bennewitz and Rotger, and Wegel and Walther in a frequency range between 10^{-1} and 10^5 cycles per second. Another calculated curve shows the loss angle of lead glass, in its dependence upon temperature between 20 and 140 C. The results are consistent with experimental data on the dielectric loss of glasses obtained by Strutt.

Fig. 1 Current density vs. field strength for a plastic dielectric, calculated after Eq. (6).

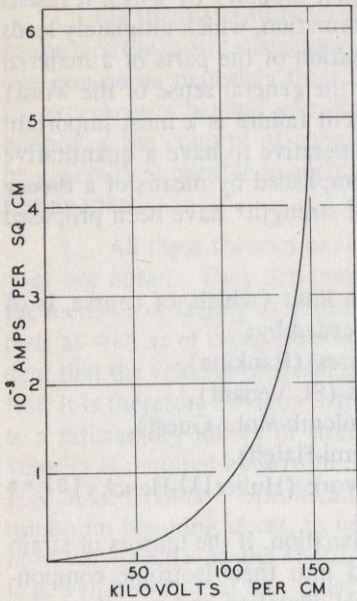


Fig. 3 Loss angle vs. temperature at low frequencies for lead glass,

- A: **calculated** after Eq. (7) from solid friction data of Gemant a. Jackson,
- B: **experimental** data of Strutt.

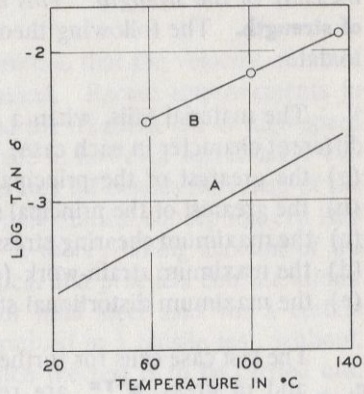
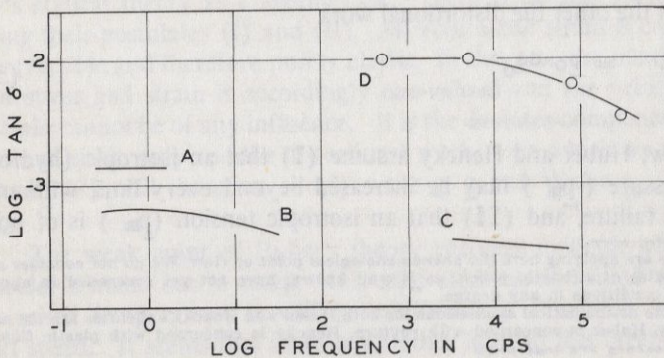


Fig. 2 Dielectric loss angle vs. frequency at room temperature for soda glass.

- A, B, C: **Calculated** after Eq. (7)
 - A) Gemant a. Jackson
 - B) Bennewitz a. Rotger
 - C) Wegel a. Walther.
- D: **experimental** data of Strutt.



A THERMODYNAMIC THEORY OF THE STRENGTH OF MATERIALS.

by M. Reiner (Jerusalem) and K. Weisenberg (Southampton).

1. The strength of a material is that property by which it resists either ruptures or excessive plastic deformation, which ultimately leads to rupture. Rupture is a visible separation of the parts of a material body. When the body is strained (in the general sense of the word) beyond its strength, it **fails**. To prevent failure is a most important task of engineering, which makes it imperative to have a quantitative measure of the strength. This is accomplished by means of a **theory of strength**. The following theories of strength* have been proposed to date:

The material fails, when a certain limit (which, of course, is of different character in each case) is exceeded by:

- (a) the greatest of the principal stresses (Rankine),
- (b) the greatest of the principal strains (St. Venant),
- (c) the maximum shearing stress (Coulomb-Mohr-Guest),
- (d) the maximum strain-work (Beltrami-Haigh),
- (e) the maximum distortional strain-work (Huber(1)-Hencky(2)**)

The last case calls for further explanation. If the tensors of strain $\underline{\underline{e}}$, and of stress, $\underline{\underline{p}}$, are resolved into their isotropic components $\underline{\underline{e}}_v$ and $\underline{\underline{p}}_m$, where $\underline{\underline{e}}_v$ is the cubical dilatation and $\underline{\underline{p}}_m$ the mean tension, and their deviator components, $\underline{\underline{e}}_o$ and $\underline{\underline{p}}_o$, the elementary strainwork

$$dW = \underline{\underline{p}} \cdot d\underline{\underline{e}} \tag{1}$$

can likewise be resolved into two components, of which the first is the work of dilatation or the volumetric work

$$dW_v = \underline{\underline{p}}_m \cdot d\underline{\underline{e}}_v \tag{2}$$

and the other the distortional work

$$dW_o = \underline{\underline{p}}_o \cdot d\underline{\underline{e}}_o \tag{3}$$

Now, Huber and Hencky assume (I) that an isotropic (hydrostatic) pressure ($-\underline{\underline{p}}_m$) may be increased beyond every limit without causing failure, and (II) that an isotropic tension ($\underline{\underline{p}}_m$) is of no influ-

* We are applying here the phenomenological point of view. We do not consider atomistic theories of strength, which, as is well known, have not yet succeeded in approaching real conditions in any degree.

** The mathematical expressions for both Huber and Hencky's theories are the same, but while Huber is concerned with rupture, Hencky is concerned with plastic flow.

*** vectors are underlined

ence upon plastic flow but may cause failure by exceeding the true molecular cohesion of the material. If the latter is not exceeded, the material may fail by W_0 exceeding a limit of maximum distortional work.

Of the theories enumerated above, Huber's and Hencky's theory has proved the most satisfactory. For homogeneous or quasi-homogeneous **non-porous** materials their assumption (I) has been established by experiments beyond any doubt, while (II) is a logical consequence of the concept of cohesion. Their theory has been shown to be generally applicable to such tests, as are carried out at the ordinary testing speeds.

2. All these theories have in common that the velocity of strain does not enter. They are purely **statical**. Recent improvements in the methods of testing of materials and the frequent use of high-speed tests as well as of creep tests have made it, however increasingly evident that the velocity of strain considerably influences the result of the test. It is therefore obvious that no purely statical conception can lead to a satisfactory theory of strength. A theory taking account of the velocity is required both from theoretical and practical considerations. E.g. Most Standard Specifications for mild steel asks for a certain minimum breaking stress, to be determined in a tensile test, without, however, specifying the speed at which the test is to be carried out. It is well known that less conscientious makers of rolled steel, if their product is not up to standard at ordinary speeds, resort to the method of increasing the speed, with the result that a higher breaking stress is obtained. On the extreme end of a series of all sorts of materials, where the influence of testing-speed makes itself felt stands e.g. artificial silk, where the concept of strength has a meaning at all only in respect of a maximum velocity of strain.

Because of the satisfactory evidence for the Huber-Hencky theory at low speeds, a good dynamical theory would have to contain Huber-Hencky's statical theory as a special case. There is no necessity of modifying their postulates (I) and (II). All volumetric strain is completely reversible and therefore purely elastic. In this case the relation between stress and strain is accordingly **one-valued** and the velocity of the strain cannot be of any influence. It is the **deviator**-components of stress and strain only and the distortional work, with which we are concerned. We will therefore simply write \underline{e} , \underline{p} and W for \underline{e}_0 , \underline{p}_0 and W_0 .

3. The weak point of Huber's theory becomes evident, when we consider a metal bar in creep. If a mild steel bar is loaded at a certain raised temperature, it will be continuously elongated at a constant speed. It seems that there is no other limit to this creeping

than the appearance of heterogeneities, when the section thins down to such a width that the constituents of the material do not average any more over the cross section. The strain work performed by the load therefore also increases continuously and practically without any limit. This would mean that such a mild steel bar is of infinite strength. This, however, is, by common experience, not the case. Hencky has pictured the maximum distortional work which can be applied upon the body, or the maximum distortional energy which can be "poured" into it, without causing failure, as the contents of a vessel, which overflows at failure. Accordingly the Huber-Hencky theory would require the "vessel" for such a mild steel bar to be of "infinite contents."

Instead of a vessel of infinite contents, a "leaking vessel" would do the same trick of admitting very large and in certain cases unlimited quantities of strain work. Such a model has been proposed by one of us (3). In creep the strain work is not stored up in the form of potential elastic energy, but is dissipated. The leakage of the vessel corresponds to the dissipation of energy. Rupture would still occur when the vessel overflows, but this would not depend on the strain work performed, but on the potential energy stored up, which in general is only a fraction of the former.

4. This makes it clear that a dynamical theory of strength can only be a thermodynamical theory. One of us (4) developed a mechanics of deformable bodies founded on thermodynamics. This will be used as the basis for a thermodynamical theory of strength. Let W be the strain work, F the free intrinsic* and energy and D the bound (dissipated) energy, all per unit volume, then the first law of the thermodynamics requires

$$\frac{d}{dt} \left(\frac{W}{\rho} \right) + \frac{d}{dt} \left(\frac{F}{\rho} \right) + \frac{d}{dt} \left(\frac{D}{\rho} \right) = 0 \quad (4)$$

where ρ is the density. The density has to be introduced because the first law does not refer to unit volume, but to unit mass.

Now let us consider isothermal processes only, i.e. the heat into which the dissipated energy is converted has to be conducted away without delay. Then in accordance with the second law we have

$$\frac{d}{dt} \left(\frac{D}{\rho} \right) \geq 0 \quad (5)$$

where the sign $=$ stands for reversible and the $>$ sign for irreversible processes.

From the fact that $\frac{d}{dt} \left(\frac{D}{\rho} \right)$ vanishes for all reversible processes

* The free intrinsic energy is the total free energy minus the kinetic energy.

there follows that intrinsic free energy is equivalent to a generalized elastic potential ϕ .

If we consider the volumetric strain work separately and the quantities in equations (4) and (5) refer to the distortional work only. ρ can be taken as constant and cancels out.

Using Newton's dot for differentiation in respect of time we accordingly write

$$\dot{W} + \frac{d\phi}{dt} + \dot{D} = 0 \quad (6)$$

where it is understood that \dot{D} can only be either positive or zero.

The distortional work can be expressed in terms of stress and velocity of strain as

$$dW = - \underline{p} \cdot \underline{\dot{e}} dt^* \quad (7)$$

from which we get for the distortional power

$$\dot{W} = - \underline{p} \cdot \underline{\dot{e}} \quad (8)$$

For purely mechanical processes ϕ and \dot{D} are functions of stress and strain and their differential quotients in respect of time only. Indicating by μ and ν the order of the differential quotient, where μ and ν can have any value 0, 1, 2

∞ , we ultimately write

$$- \underline{p} \cdot \underline{\dot{e}} + \frac{d(\rho \phi \underline{p} \underline{e}^\nu)}{dt} + D(\underline{p}, \underline{e}) = 0 \quad (9)$$

This equation has been called the mechanical equation of the state of the material. 4. The functions ϕ and \dot{D} are of different form for every group of materials and the materials themselves can be distinguished by the various values of the constants which appear in the equation of state. These constants are the mechanical material constants. E.g. one group of materials is formed by all perfectly elastic bodies, which follow Hooke's law, the different materials being distinguished by the value of the modulus of rigidity γ . Another group is formed by all simple viscous liquids, which obey Newton's law, and the different liquids are distinguished by the value of the coefficient of viscosity η .

5. On the basis of the development of the preceding section we can say that **failure depends upon a maximum value of the intrinsic free energy**, i.e. the energy ϕ which can elastically be stored up in the volume-element of the material. Failure will occur when

*The negative sign is used because in Equation (6), W is the power of the external forces

$$\phi_T = \int_0^T (\dot{W} - \dot{D}) dt > K \quad (10)$$

where K is material constant, which may be called the strength modulus. Equation (10) may be called the rupture condition. Where there is no dissipation of energy or where the dissipation of energy is negligible, as in the case of a steel tensile test at **ordinary speed**, \dot{D} either vanishes or can be neglected and the condition for rupture becomes

$$\int_0^T \dot{W} dt = \phi_T > K \quad (11)$$

which is the condition of Huber and Hencky's theory. In an irreversible process where \dot{D} is in accordance with Equation (5) positive, part of the **power** of distortion leaks out and a part of it only can be stored up. At certain states of stress and strain, when $\dot{W} = \dot{D}$, the body can be strained at infinitum without failure.

In order to evaluate equation (10) for a special case, the following have to be known, viz.

- (I) the mechanical equation of state of the material,
- (II) the mechanical conditions of the test

Ad (I): this can generally be expressed as a relation between the time-differential equations of the stress and strain, or

$$R(\overset{\mu}{p}, \overset{\nu}{e}) = 0 \quad (12)^*$$

Ad (II): these consists of the dynamical and kinematical boundary conditions.

From (I) the free (potential elastic) power $\dot{\phi}$ can be calculated and by means of (II) $\dot{\phi}$ can be segregated up to breaking strength. 6. The present theory can find its verification only by comparison with actual tests. The difficulty of using existing test-results lies in this that the observations of cases of rupture and its correlation with loading, straining and velocity of strain of testpieces i.e. all knowledge required under (II), is of little value if we do not, at the same time know in accordance with the requirement (I) the mechanical equation of state of the material. This, however, has seldom, if ever, been completely determined in connection with experiments on strength. In order to check the present theory special experiments would therefore have to be undertaken. Until this is done, the present theory can only be **qualitatively** checked from known strength tests. The theory would gain very much in probability, if it could be shown that an increase in breaking strength with increasing velocity of straining, as men-

* It should be kept in mind that this is a tensor-equation.

tioned in Section 1, would follow from it. On the other hand the theory could be discarded at once, if this was not the case.

As we do not know the actual equations of state of the materials under test we have to make assumption about them. If we specialize Equation (12) for cases where only the first differential quotient appears, i.e. if we assume the equation of state to be of the form

$$R(\underline{p}, \underline{\dot{p}}, \underline{e}, \underline{\dot{e}}) = 0 \quad (13)$$

this would **qualitatively** cover all conditions. We can go a step further by assuming R to be linear, or of the form

$$C + C_2\underline{p} + C_3\underline{\dot{p}} + C_4\underline{e} + C_5\underline{\dot{e}} = 0 \quad (14)$$

If the actual equation at state would show itself quantitatively but not quantitatively.

Equation (14) can be considered as composed of two equations

$$A_1 + A_2\underline{p} + A_3\underline{e} + A_4\underline{\dot{e}} = 0 \quad (15)$$

and

$$B_1 + B_2\underline{p} + B_3\underline{\dot{p}} + B_4\underline{\dot{e}} = 0 \quad (16)$$

and accordingly the material which has Equation (14) as its equation of state, as composed of two materials, which have the equations of state (15) and (16) respectively. The first can be called an **e**-body and the second a **p**-body.

We can neglect A_1 , which can be interpreted as an initial stress and B_1 , which can be interpreted as an initial strain and equations (15) and (16) can be written as

$$\underline{p} - 2 \gamma_s \underline{e} - 2 \eta_s \underline{\dot{e}} = 0 \quad (17)^*$$

$$2\underline{\dot{e}} - \underline{p}/\eta_z - \underline{\dot{p}}/\gamma_z = 0 \quad (18)$$

As can be easily seen, the first of these bodies is a solid, the second a liquid. The constant γ is an elastic modulus and the constant η a coefficient of viscosity. The subscripts s and z are meant to indicate the solid and the liquid state and η_s is accordingly the measure of the viscous resistance to the deformation in a solid, while γ_z is the measure of the elasticity of a liquid. Where η_s is absent, the first body is

* The factor 2 results from the usual definition of the material constants γ and η , or, what comes to the same thing, of shear.

reduced to a Hookean solid, where γ_z is absent the second body is reduced to a Newtonian liquid. The ideal material represented by equation (17) may be called a Voigt-solid (5), the other represented by (18) may be called a Maxwell-liquid (6). While it is not maintained that any real materials could be perfectly represented by either a Hooke or a Voigt-solid or by a Newtonian or a Maxwellian liquid, it may be assumed that the properties of the ideal materials represented by the equation of state (17) and (18) can be encountered at varying degrees in the mechanical behaviour of every real material.

7. (a) In the Hooke-solid, the equation of state of which is

$$\underline{p} = 2\gamma e \quad (19)$$

all the strain-power of the external forces is converted into elastic potential energy and none is dissipated, or

$$\frac{d\phi}{dt} = \underline{p} \cdot \dot{e} = 2\gamma e \dot{e} = \gamma \frac{d}{dt} e^2 \quad (20)$$

which gives

$$\phi = \gamma e^2 = \frac{p^2}{4\gamma}; \quad \dot{D} = 0 \quad (21)$$

(b) In a Newtonian liquid the equation of state of which is

$$\underline{p} = 2\eta \dot{e} \quad (22)$$

all the strain power of the external forces is dissipated, or

$$\phi = 0, \quad \dot{D} = \underline{p} \cdot \dot{e} = 2\eta \dot{e}^2 = \frac{p^2}{2\eta} \quad (23)$$

The function \dot{D} is called Raleigh's dissipation function.

(c) For the Voigt-solid equation (17) gives

$$\underline{p} \dot{e} = 2\gamma_s e \dot{e} + 2\eta_s \dot{e}^2 \quad (24)$$

and comparing Equation (24) with Equation (9) we find

$$\frac{d\phi}{dt} = 2\gamma_s e \dot{e} = \gamma_s \frac{de^2}{dt}; \quad \phi = \gamma_s e^2, \quad \dot{D} = 2\eta_s \dot{e}^2 \quad (25)$$

(d) For the Maxwell liquid Equation (18) gives

$$\underline{p} \dot{e} = \frac{p^2}{2\eta_s} + \frac{p\dot{p}}{2\gamma_z} \quad (26)$$

and comparing Equation (26) with Equation (9) we find

$$\frac{d\phi}{dt} = \frac{p\dot{p}}{2\gamma_z} = 1/4 \gamma_z (d p^2/dt) \quad (27)$$

$$\phi = p^2/4\gamma_z, \quad \dot{D} = p^2/2\eta_z$$

Applying our rupture condition Equation (10) to these four cases, we find:

(a) A solid, which is perfectly elastic up to the yield point and is therefore an ideal plastic material, or which is perfectly elastic up to rupture and is therefore an absolutely brittle material, fails through plastic deformation in the first or through rupture in the second once,

when either the stress or the strain reaches a certain limit. For such materials our criterion is therefore reduced to Hencky's and Huber's criteria respectively, and these in their turn reduced to Rankine's and St. Venant's criteria respectively, which give in this case identical results, provided the **deviator-components** of the stress and strain-tensors are taken and not the complete tensors.

(b) A simple viscous liquid, which had no elasticity, can only be "broken up" by hydrostatic tension. In laminar flow, where the mean pressure is equal to the hydrostatic pressure, a simple viscous liquid **cannot** fail, because for such a material $\dot{\nu}$ vanishes and is therefore smaller than any limit which may be put to its strength.

(c) A Voigt solid fails when the **strain** reaches a certain limit. There is no one-valued relation between stress and strain and no definite limit of stress corresponds to the limit of strain. Let \underline{e}_f be the strain failure, then equation (17) shows that

$$\underline{p}_f = 2\gamma \underline{e}_f + 2\eta_B \dot{\underline{e}} \quad (28)$$

and that the stress at which the material fails increases with the velocity of strain. Let p be the stress at which the material fails in a static test, i.e. with vanishing strain-velocity, then there is

$$\underline{p}_f = p + 2\eta_B \dot{\underline{e}} \quad (29)$$

As η_B can be determined from the damping of free oscillations of the material under test, i.e. independently of the strength test, the dependence of the strength of the material from the speed of deformation can be predicted in accordance with Equation (29). It is not maintained that this dependence will in the case of say a mild steel bar, actually be a linear one. On the contrary, it may be expected that it will not be linear, because it is well known that equation (17) does not correctly describe the damping of free oscillations of a metal wire. If however from oscillation experiments the correct form of the functional relationship which had to replace Equation (17) is determined, there is no difficulty in deriving the relationship, which had to replace Equation (29). Qualitatively, however, it is shown that our theory gives the result that **the failure-stress of an elastic material, which shows damping of its free oscillations, increases with increasing velocity of strain.**

(d) A Maxwellian liquid, i.e. a viscous liquid which shows elasticity, on the other hand fails when the **stress** reaches a certain limit \underline{p}_f . To this limit there does **not** correspond a definite limit of the velocity of strain, Equation (18) shows that

$$\dot{\epsilon}_f = \underline{p}_f / 2\gamma_2 + \underline{p} / 2\gamma_1 \quad (30)$$

Let $\dot{\epsilon}$ be the velocity of strain at which the material fails if the stress is applied infinitely slow, then there is

$$\dot{\epsilon}_f = \dot{\epsilon} + \underline{p} / 2\gamma_1 \quad (31)$$

The modulus of elasticity γ_2 can be determined oscillation-experiments (7), and the maximum velocity of strain at which failure occurs predicted in accordance with Equation (31). This is applicable to failure of a metal or any other material in creep (e.g. concrete), which can be considered a very viscous and elastic liquid and of such material as artificial silk, etc., or all liquids, which are "spinnbar." Here also Maxwell's linear relation is too simple (8)) and the correct equation of state had to be determined for every group of materials by experiment.

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