

# RHEOLOGY BULLETIN

Publication of the Society of Rheology

Volume 30, No. 2

Fall, 1961

## THE BINGHAM MEDAL, 1961

The Bingham Medal of the Society of Rheology for 1961 will be awarded to Mr. William R. Willets, of the Titanium Pigment Corporation, New York, N. Y. The recipient was chosen by the Bingham Award Committee consisting of the following members: E. B. Bagley, J. T. Bergen (Chairman), W. F. Fair, R. R. Myers and J. P. Tordella.

William R. Willets was born in 1905 in Montclair, N. J., and attended the State College of Forestry, Syracuse, N. Y., where he received the B.S. degree in 1926. He was then employed by the Oxford Paper Company, and in 1929 he joined the Western Electric Company to work on pulp insulation for telephone cables. Since 1933 he has been employed by the Titanium Pigment Corporation where he is presently Assistant Manager of the Technical Service Laboratories. During the years 1942 to 1944 he served as a Consultant to the War Production Board on conservation of paper resources for World War II.

His professional activities have covered much of the field of paper technology. During the past thirty years he has been the author or co-author of over 50 articles and technical papers. For the latter half of that period his interest has included the rheology of paper-making and paper coatings. His work in this connection has contributed to theories of filler retention and has helped to acquaint the paper industry with rheological concepts.

A member of the Society since 1946, Mr. Willets served as Chairman for Local Arrangements for Society Meetings in New York in 1949 and 1950. In 1951 he was the Society's representative on the A.I.P. Committee for the Twentieth Anniversary Joint Meeting in Chicago. Elected Secretary-Treasurer in 1953, he has continued in that office until the present. During this time he was largely responsible for the establishment of the good relationship which now exists between the Society and The American Institute of Physics. He was instrumental in setting up a sound working basis between the Society and the Interscience Publishing Company. His careful stewardship of the Society's funds has resulted in a strong financial position for the years ahead.

Mr. Willets is also a member of the Governing Board of A.I.P., a member of the Board of Directors of A.S.T.M., Chairman of the Pigments and Fillers Testing Committee of the TAPPI, and a Fellow of the American Institute of Chemists.

## THIRTY-SECOND ANNUAL MEETING 1961

The Annual Meeting of the Society of Rheology will be held this year at the University of Wisconsin at Madison, Monday, October 30 through Wednesday, November 1. The eight technical sessions (including one on Monday evening), as well as the smoker on Tuesday evening when the Bingham Medal will be presented, will be in the Wisconsin Center Building at the University.

*Meals.* For the convenience of members, lunch will be served at the Wisconsin Center on Monday, Tuesday, and Wednesday, and dinner on Monday. It will be necessary to obtain tickets either by preregistering or by purchase upon arrival.

*Hotel.* A block of rooms has been set aside at the Hotel Loraine and there are several other hotels in Madison. Members of the Society will receive a registration card for hotel accommodation.

*Registration.* Registration cards for the Meeting will be sent to all members to provide an opportunity of preregistering by mail. The cards should be forwarded to Professor A. M. Swanson, Department of Dairy and Food Industry, University of Wisconsin, Madison 6, Wisc. as requested in the circular about the Meeting to be forwarded to each member. You will be able to pick up your registration badge and tickets at a desk which will be located Sunday evening, October 29 at the Hotel Loraine and Monday morning and thereafter at the Wisconsin Center Building.

The details of the meeting are in the hands of the Local Arrangements committee: Professors John D. Ferry (Chairman), Millard W. Johnson, Jr., and Arthur M. Swanson.

## PROGRAM OF MEETING

Technical sessions will be held in the auditorium of the Wisconsin Center, apart from Session II on Wednesday afternoon, which will be held in Room 210 of the Wisconsin Center.

In view of the large number of papers submitted for presentation at the 32nd Annual Meeting, it was necessary for the Program Committee to schedule an evening session. A symposium on "Blood Rheology" was selected because it was believed to be of sufficient interest to attract a sizable audience. Arrangements have been made to hold this symposium on Monday evening, October thirtieth, 8:00 to 10:00 p.m.

## RHEOLOGY BULLETIN

E. H. Lee, Editor

Division of Applied Mathematics

Brown University

Providence 12, R. I.

The other technical sessions occur on the mornings and afternoons, Monday, October 30th to Wednesday, November 1st inclusive, with two simultaneous sessions on Wednesday afternoon, November 1st.

The Bingham Medal will be awarded on Tuesday evening, October 31st, at a Smoker at the Wisconsin Center.

Monday Morning, October 30

9:30-12:00

Opening of the Meeting: JOHN H. ELLIOTT, President of the Society.

Welcome on behalf of the University: DEAN JOHN E. WILLARD, Dean of the Graduate School

Technical Session on Birefringence. Chairman: DR. J. G. BRODNYAN

### Rheo-Optical Properties of Polymers

D. G. LEGRAND and P. F. ERHARDT, *General Electric Research Laboratory, Schenectady, New York.*

The birefringence and stress-strain properties have been measured simultaneously under constant or dynamic stress conditions as a function of temperature. Under certain conditions, there appear to be two optical constants associated with the material, which reflect the elastic and viscous response of the material. Data for polyethylene, polypropylene and polystyrene will be presented and discussed.

### Birefringent Techniques in Two-Dimensional Flow

D. C. BOGUE and F. N. PEEBLES, *University of Tennessee, Knoxville, Tenn.*

A combination of the flow birefringent technique, which has been developed for Newtonian fluids, with the stress optical laws for concentrated polymers appears to offer a powerful means of studying two-dimensional viscoelastic flow. Provided that certain generalizations of the stress optical laws are made and that normal stresses are measured at the boundaries, one can in principle obtain a complete stress description without resort to a rheological model. This paper is concerned with a description of the proposed method and a discussion of its experimental feasibility. In addition some experimental results for two-dimensional flows with a dilute birefringent material will be discussed.

### A Method of Measurement of Flow Birefringence of a Thin Fluid Layer for Oscillatory Shear\*

GEORGE B. THURSTON and JOHN L. SCHRAG, *Physics Department, Oklahoma State University, Stillwater, Oklahoma.*

A method of measurement of oscillatory flow birefringence of a thin fluid layer has been developed. The fluid layer is that confined between two rigid parallel planes, one plane being fixed and the other plane executing a sinusoidal motion. A thin sheet of light traverses the fluid layer parallel to the planes. The light source is stroboscopically synchronized with the fluid motion, and is controlled so that the optical effects may be determined for all time epochs during the cycle of motion. Measurements have been carried out using aqueous milling yellow solutions in the frequency range from 10 to 130 cps. It is found that a variable phase relation exists between the birefringence and the sinusoidally time varying motion. Numerical factors have thus been obtained for the magnitude and phase of a complex mechano-optic factor describing the flow birefringence, this factor being defined as the complex ratio of the amount of birefringence to the velocity gradient.

\*This work was supported by the Office of Ordnance Research of the U. S. Army

### Correlation of Static and Dynamic Measurements Using Flow Birefringence

S. J. GILL, *University of Colorado, Boulder, Colo.*, and W. PHILIPPOFF, *Esso Research & Engineering, Linden, N. J.*

The correlation of steady-state and dynamic measurements in rheology is still not directly possible. However, recently a new device using flow birefringence, a technique usually applied to steady-state measurements, has been designed which allows dynamic investigations. Three solutions have now been investigated using this new device together with the usual flow birefringence and viscosity measurements. The solutions used are a 1.8% carboxymethyl cellulose in water, a 30% solution of polystyrene in toluene, and a 4% solution of Vistanex in a hydrocarbon oil. All these solutions have about the same "Maxwell constant." The results show how the new device can be correlated with existing methods.

### Correlation of Ultrasonic Experiments and Steady State Measurements on Polymer Solutions in Oil

W. PHILIPPOFF, *Esso Research and Engineering, Linden, N. J.*

One basic problem in Rheology is the correlation of steady state and dynamic measurements. At present a large volume of experimental results exists for both types of experiments without an explicit formula existing to connect them. The difficulty consists in the fact that in steady state measurements, there is basically a non-linear differential equation between stress and strain, whereas in dynamic measurements (vibration)

it is experimentally known that stress and strain are proportional to each other; in other words, that the so-called linear elasticity is applicable.

From basic mechanics the correlation exists that the rate of shear  $D = 4\Pi f$  where  $f$  is the frequency of the vibrations. Existing experiments in the same range of  $D$  and  $f$  show that this correlation is not valid: the vibrational technique causes a much larger change in viscosity than the steady flow technique at  $D = 4\Pi f$ . We have had an opportunity to measure a 3% solution of polyisobutylene in white oil in numerous previous publications. Both flow birefringence and ultrasonic measurements are collected for a large range. The correlation showed that viscosities cannot be compared at  $D = 4\Pi f$  but rather with the constant being about 0.1 instead of  $4\Pi$ .

Further a comparison of the recoverable shear  $s$  from the capillary length effect and from jet experiments calculated according to Metzner's formula, with  $G''/G' = \tan(\delta)$  in the ultrasonic experiments was made. In a previous publication it has been pointed out that only on this basis could one find a correlation of the elastic properties. At the same frequencies where previously the viscosities correlated, both values were comparable. By such empirical methods it is possible, lacking the theory, to obtain an insight into the proper correlation.

Monday Afternoon, October 30

1:30-5:00

*Technical Session on Viscoelasticity. Chairman: PROFESSOR J. D. FERRY*

#### Dynamic Measurement of Mechanical Properties of Polymers by Free Vibration Methods II. The Glass Transition Concentration

ARMAND F. LEWIS and MARVIN C. TOBIN, *Chemical Research Department, Central Research Division, American Cyanamid Company, Stamford, Conn.*

Results of measurements on the dynamic mechanical properties of cotton thread filled polymer solutions are reported. It is found that as the solvent evaporates, the apparent dynamic flexural modulus goes through an inflection while the damping goes through a maximum. The concentration (volume fraction of polymer) at which this happens is defined as the "glass transition concentration." It is suggested that on a molecular scale, the glass transition occurs when the solvent concentration gets low enough so that free rotation of the polymer chain segments is restricted. Experimental results are reported for a series of poly(methacrylates) and polystyrene in various solvents. These data are compared with the dynamic mechanical properties of the cotton-filled polymers at various temperatures. The relation between the glass transition concentration and glass transition temperature is discussed.

#### Solution and Experimental Results for a Problem in Linear Viscoelasticity

J. H. BALTRUKONIS, *Catholic University of America, Washington, D. C.*, and —

W. G. GOTTENBERG and R. N. SCHREINER, *Space Technology Laboratories, Inc., Los Angeles, California.*

A solution is obtained for the problem of the axial vibrations of a rigid circular rod embedded in a linear, viscoelastic material which is, in turn, contained within a rigid circular casing. It is assumed that the entire assembly is infinitely long. The response of the embedded rod to steady-state, harmonic oscillation of the casing is calculated within the framework of the small deformation theory of linear viscoelasticity. This result is applied in devising an experimental method to measure the complex shear modulus of the embedding, linear viscoelastic material as a continuous function of frequency. Experimental data are presented to illustrate the method and to demonstrate some of the associated difficulties. The most difficult requirement to be satisfied experimentally is that the assembly behave as if it were infinitely long. An interesting empirically determined relationship is shown between the necessary length and the radial geometry of the system.

#### Viscoelastic Properties of Dilute Polystyrene Solutions and Verification of the Zimm Theory

RICHARD B. DEMALLIE, JR., MEYER H. BIRNBOIM, J. E. FREDERICK, N. W. TSCHOEGL, and JOHN D. FERRY, *University of Wisconsin, Madison, Wis.*

Storage ( $G'$ ) and loss ( $G''$ ) shear moduli have been measured over a wide frequency range with the apparatus of Birnboim and Ferry for dilute solutions of a polystyrene with sharp molecular weight distribution,  $M_w=267,000$ , in Aroclor 1248, a chlorinated diphenyl. The high viscosity of the solvent (2.2 poise at 25°C.) ensured that the viscoelastic dispersion fell within the experimental frequency region. The concentration range was 0.5 to 3% and the temperature range from 0° to 40°C. The results did not follow the theory of Rouse but were in close accord with the theory of Zimm, as follows: (a) the ratio  $(G'' - wn_s)/G'$ , where  $w$  is circular frequency and  $n_s$  solvent viscosity, agreed with the theoretical value of 1.73 at higher frequencies; (b)  $G'' - wn_s$  and  $G'$  were proportional to  $w^{2/3}$  in this region; (c) the experimentally determined terminal relaxation times agreed with those calculated from the solution viscosity within experimental error; (d) the molecular weights calculated from the Zimm theory were of the correct magnitude, though somewhat too high at the higher concentrations. At 3% concentration, some divergence from the Zimm theory appears at high frequencies. The sharp molecular weight distribution and the solvent viscosity, which should minimize effects of internal viscosity of the polymer chain, are probably important in achieving the good agreement with the theory.

## Stress Relaxation and Dynamic Mechanical Properties of Stretched Natural Rubber

EDWIN R. FITZGERALD,\* ANTHONY J. BUR\* and EDWARD A. METZBOWER\*, *Department of Physics, The Pennsylvania State University, University Park, Pennsylvania.*

Stress relaxation measurements have been carried out on the same "pure gum" natural rubber stock on which dynamic mechanical measurements have previously been made.<sup>1</sup> The stress relaxation of a single sample was followed for 48 hours at each of 8 successive elongations between 0 and 275% at 25°C. Stress-strain curves at 10, 24 and 48 hours derived from these data all show a definite shoulder or minimum between 180 and 200% elongation in agreement with the results of Martin, Roth and Stiehler<sup>2</sup> from creep measurements on a simple pure gum rubber stock. This minimum apparently corresponds to the maximum in the dynamic shear compliance previously found at 185% elongation and indicates that the dynamic measurements are related to the slope of the static stress-strain curve for elongations below 300%. Equivalent creep curves were also calculated from the stress relaxation data and the rate of creep vs elongation (or initial stress) determined.

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<sup>1</sup>E. R. Fitzgerald, *J. Acoust. Soc. Am.* (to be published, 1961).

<sup>2</sup>G. M. Martin, F. L. Roth and R. D. Stiehler, *I.R.I. Transactions* 32 189 (1956).

## Viscoelastic Behavior of Amorphous Elastomers Subjected to Large Tensile Deformations at Constant Strain Rates

THOR L. SMITH, *Stanford Research Institute, Menlo Park, California.*

When viscoelastic behavior is linear, stress-strain curves measured at various constant rates of strain superpose to form a single curve on a plot of  $\log s(e, t)/e(t)$  vs  $\log t$ , where  $s(e, t)$  is the stress, which is a function of strain  $e$  and time  $t$ , and  $e(t)$  is the strain which is directly proportional to time. The ratio  $s/e$  is a function only of time and can be called the constant-strain-rate modulus  $F(t)$  which is related exactly to the stress-relaxation modulus  $E(t)$  by the equation  $E(t) = F(t) (1+m)$ , where  $m = d \log F(t)/d \log t$ .

Stress-strain curves of amorphous elastomers measured under non-equilibrium conditions out to large deformations are curved because of both time effects and the inherent non-linearity of such materials. Often, these factors can be separated by defining the modulus  $F(t) = g(e) s(e, t)/e(t)$  where  $g(e)$ , a function only of strain, approaches unity as  $e$  approaches zero. An analysis was made of stress-strain curves of an SBR gum vulcanizate measured to rupture at numerous strain rates at temperatures between -43 and 88°C. From -34 to 88°C,  $g(e)$  was found to be independent of both time and temperature, but at -43°C and for

strains greater than 0.75,  $g(e)$  was found to be different than at the higher temperatures. The function form of  $g(e)$  was compared with that predicted by several analytical expressions for representing stress-strain data. To show further the advantages of using  $F(t)$  in analyzing time-dependent stress-strain curves, previously published data (*J. Polymer Sci.* 20, 89 [1956]) on the NBS polyisobutylene were analyzed by the new procedure and  $E(t)$  was calculated from the composite plot of  $\log F(t) 298/T$  vs  $\log t/a_T$ .

## Large Longitudinal Retarded Elastic Deformation of Rubberlike Network Polymers

HERBERT LEADERMAN, *National Bureau of Standards, Washington, D. C.*

Measurements have been made of the creep under constant tensile load and creep recovery following removal of load up to a relative length  $l$  of about 1.25 on a specimen of plasticized polyvinyl chloride; similar measurements have also been made of the creep under constant tensile stress. It was found that the relative length as a function of time was given by the one-dimensional linear superposition equation of Boltzmann relating response to previous excitation history. If the nominal tensile stress is taken as the excitation, a suitable measure of response is  $(l - l^2)/3$ . If the actual tensile stress is taken as the measure of excitation, a suitable measure of response is  $(l^2 - l^{-1})/(1 + k l^{-1})/3(1 + k)$  with  $k$  equal to 0.8.

Monday Evening, October 30

8:00-10:00

*Symposium on Rheology of Blood.*

*Chairman:* DR. P. S. FRANCIS

## Special Problems of Blood Rheology<sup>1</sup>

MELVIN H. KNISELY, PH.D., *Department of Anatomy, Medical College of South Carolina, Charleston, South Carolina.*

The essential purpose of studying the rheology of blood is to determine its rheological behavior within the living body. In health, blood cells are not agglutinated (Kniseley, Warner and Harding, 1960). Red cells are known to carry small negative electric charges (Abramson, 1934) which cause them to repel each other slightly. To carry oxygen to tissues blood cells must pass through the true terminal arterioles, arterial capillaries and capillaries or sinusoids of an organ. In health unagglutinated mammalian blood flows easily and rapidly through terminal arterioles having diameters down to 12, 10, or at times even 5 micra. The rheology of unagglutinated blood must be studied separately from that of sludged blood from diseased subjects. In preparing a review we found records of 7,956 patients

having 162 different diseases in which the red blood cells were agglutinated into wads and masses of various sizes. Many investigators record observing that such masses of blood cells were seen to plug terminal arterioles temporarily, for short periods, or permanently. Also, in man and animals blood-cell masses have been seen to settle out and remain stationary on the lower sides of vessels during the life of the subject (Harding and Knisely, 1958; Knisely, Warner and Harding, 1960; Knisely, 1961). The whole circulatory system acts as a sieve and a settling tank which continually remove from the circulating blood masses above certain sizes. Obviously blood-cell masses which plug terminal arterioles or remain on the lower sides of vessels cannot be taken into a needle inserted into a vein to obtain a sample of blood for rheologic studies (Knisely, 1960). Such samples from patients having agglutinated blood can never tell the whole story of their blood rheology.

Because blood-cell masses which settle to the lower sides of vessels may later become suspended, the concentrations of blood-cell masses in different parts of the circulatory system are not necessarily the same. One set of rheologic phenomena can be going on in one part of the vascular system and others in other parts.

Equations developed for expressing rheologic ideas often are a result of assuming that blood vessels are cylinders. In man and experimental animals the aorta and all segments of arteries between branches are not cylinders, but long slowly tapering truncated cones. Many segments of veins are cone shaped. Apparently flow through cones has not been taken into account in the studies of blood rheology.

Often the capillary tube viscometer or rotating cup viscometer used to measure the properties of blood has spaces for the sample much wider than individual red cells or of blood-cell masses. Such instruments do not determine the resistance of blood to flow through the narrowest vessels.

Hence, some problems are: How can we make instruments to determine and measure the physical properties of unagglutinated and agglutinated blood? How can we take steps to understand the flow through segments of truncated cones? How can we take realistic steps to get proper samples of pathologic blood from different parts of the body for rheologic studies?

<sup>1</sup>Supported by United States Public Health Service Grant H-4176.

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#### The Rheology of Blood in the Living Microvascular System

EDWARD H. BLOCH, M.D., PH.D., *Department of Anatomy, Western Reserve University, Cleveland, Ohio.*

Blood flow was studied in 20 to 500 micra arterioles and venules with high speed (to 7,600 frames per second (fps)) and standard (16-24 fps) cinephotomicrography, and by microspectrophotometry. Also an image orthicon television system was combined with microspectrophotometry and the images recorded from the monitor. Frogs, rats, and rabbits were used.

In the average anesthetized animal blood flow in arterioles and venules (diameters greater than 30 micra) was so rapid when studied by standard methods that the blood stream appeared as a reddish column with a lighter central streak (in some vessels) and often with a clear cell free zone adjacent to the luminal wall of the vessel. In venules a similar clear zone also appeared between columns of confluent blood streams. Cellular detail could *not* be observed until such circulations were photographed at more than 1,500 fps (studied at projection rates of 16-24 fps). Then it was seen that the cells (erythrocytes) were randomly oriented; frequently at right angle to the direction of flow. Their pathways were essentially helical. Frequently cells rotated about their short axis and moved across "lamina". Cellular concentration varied from moment to moment as did the magnitude of the peripheral plasma layer. While the diameter of the axial stream decreased with increasing flow rates as observed by standard methods this was not borne out by the high speed film. Cellular (erythrocyte) deformation occurred in all vessels and occurred at a frequency of the order of msec. Undeformed cells were rarely observed.

#### The Comparison of the Shear Stress-Shear Rate Characteristic of Blood with the Tube Flow Behavior<sup>1</sup>

S. E. CHARM, *Department of Nutrition, Food Science and Technology, Massachusetts Institute of Technology, and —*

G. S. KURLAND, *Department of Medicine, Harvard Medical School; and Medical Research Department and Medical Service, Yamins Research Center, Beth Israel Hospital.*

The shear stress-shear rate characteristic of heparinized canine blood was determined in a cone and plate viscometer. The pressure loss and flow rate behavior of the blood was determined in glass capillary tubes. It was found that the slope of log shear stress *vs.* log shear rate curve was always lower than the slope of the log pressure loss *vs.* log rate of flow curve.

The shear stress in the cone and plate viscometer ranged from approximately 1 dyne/cm<sup>2</sup> to 15 dynes/cm<sup>2</sup> while the wall shear stress in the tubes ranged from approximately 10 dynes/cm<sup>2</sup> to 400 dynes/cm<sup>2</sup>.

This difference could possibly be explained by considering a marginal gap of fluid at the wall. In the smallest capillary tube employed (.00883 cm dia.), a moving picture of flowing blood taken through a microscope suggested a marginal gap of  $3 - 5 \times 10^{-4}$  cm.

From considerations employing the tube flow data with the cone and plate viscometer data, a marginal gap of  $3.25 \times 10^{-4}$  cm was calculated. The size of the marginal gap appeared to increase with tube diameter.

<sup>1</sup>Supported by The Massachusetts Heart Association, Inc., Grant No. 464.

### Shear Rate Dependence of Viscosity of Human Blood and Blood Plasma

ROE E. WELLS, JR., M.D., *Department of Medicine, Harvard Medical School, and Peter Bent Brigham Hospital, Boston Massachusetts*, and —

EDWARD W. MERRILL, *Department of Chemical Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts, and Peter Bent Brigham Hospital, Boston, Massachusetts*.

Viscometric studies on whole human blood, analyzed immediately after removal from the vein of the subject, and again after anticoagulants had been added show:

(1) blood treated with anticoagulants (heparin, citrate, oxalate) is non-Newtonian, viscosity decreasing with increase of shear rate.

(2) The same blood tested immediately, without anticoagulants, is far *more* non-Newtonian, with higher viscosities at the lower shear rates.

Plasma, the continuous phase of whole blood, is found to have substantial non-Newtonian rheology when analyzed immediately after the whole blood has been taken from the subject and centrifuged to remove the red cells. The non-Newtonian rheology is repressed, or eliminated altogether, by addition of anticoagulants.

It is estimated that for the major vessels of the circulation, in which it is appropriate to consider blood homogeneous, the shear rate at the wall varies from  $500 \text{ sec}^{-1}$  (in the aorta) to  $20 \text{ sec}^{-1}$  in the arterioles.

In the smallest vessel, the capillary, the diameter is only slightly larger than that of the red cells, which move through it in single file. In the film of plasma between red cells and capillary wall, the shear rate may be as high as  $1000 \text{ sec}^{-1}$  whereas between any two red cells it is negligible.

The observed rheology of freshly drawn whole blood and blood plasma is discussed in relation to flow in the vessels of the circulation. Two viscometers were used: (1) A cone-plate viscometer, operating over a range of 12 to  $250 \text{ sec}^{-1}$  of shear rate, and containing 2 ml. of sample. (2) A Couette type, designated GDM, which was designed to measure low viscosities (around 1 centipoise), over a range of low shear rates ( $0.1$  to  $20 \text{ sec}^{-1}$ ), with a small sample (3 ml.), within a period of 50 seconds.

Tuesday Morning, October 31

9:00-12:00

*Technical Session on Shear Rate Dependence.*  
Chairman: DR. T. G. FOX

### Measurement of High Viscosities at Low Shear Rates

D. C. WEST, *Central Research Laboratory, Canadian Industries Limited, MacMasterville, Que., Canada*.

In the "meniscus velocity" method, a short column of viscous fluid is placed within a vertical glass tube of radius  $R$ , open at both ends. The top and bottom surfaces gradually change shape as the core of the liquid column falls under its own weight, but the lines of contact with the glass wall do not move. The velocity of the center of the meniscus,  $v_0$ , is measured, and the viscosity is found from the simple formula  $\eta = \frac{4}{3} R^2 v_0$ , which depends on the second power of the tube radius and is independent of the sample length. Corrections for meniscus shape and surface tension are minor. The measurements are reproducible, absolute, and reasonably rapid, for materials from 12,000 to 800,000 poise. For a material of given density  $\rho$ , the shear stress is fixed by the tube radius and can be from 150 to 300 dynes/cm<sup>2</sup>. Shear rates range from .0002 to .01 reciprocal seconds.

### A New Instrument for High Shear Viscometry

DALE G. WILLIAMS, CARROLL L. GAREY, and GLEN A. HEMSTOCK, *The Institute of Paper Chemistry, Appleton, Wisconsin*.

A high shear viscometer capable of producing rheograms under conditions approaching those estimated to exist in coating equipment is described. The instrument is of a coaxial cylinder type with translational movement supplied by air pressure. The strain, measured by means of a magnet transducer, and the viscous stress, measured by means of a piezoelectric transducer, are recorded with the aid of an oscilloscope. This instrument is capable of operation to shear rates of  $1.2 \times 10^5 \text{ sec}^{-1}$  at rates of change of rate of shear of  $1.7 \times 10^7 \text{ sec}^{-2}$ . Because of the short time of operation in obtaining a rheogram, there is no unnecessary working of the test fluid and only a minimum temperature rise. Rheograms of mineral oil, glycerin and a paper coating color for shear rates up to  $6.2 \times 10^4 \text{ sec}^{-1}$  at rates of change of shear rate to  $4.7 \times 10^6 \text{ sec}^{-2}$  are discussed.

### Non-Newtonian Flow of Concentrated Solutions of High Polymers

SHIGEHARU ONOCI,\* TADASHI KOBAYASHI, YASUHIRO KOJIMA and YOSHISHIGE TANIGUCHI, *Department of Polymer Chemistry, Kyoto University, Kyoto, Japan*.

Non-Newtonian flow of concentrated solutions of polyvinyl alcohol (PVA) in water and polystyrene (PS) in toluene have been measured by means of a

Maron-Krieger-Sisko viscometer at various temperatures and concentrations. The usual dependence of the apparent and zero-shear viscosities for two polymers on rate of shear, temperature and molecular weight is found. The log-log plot of zero-shear viscosity against concentration for PVA and PS can be represented by two straight lines intersecting at one point (critical concentration  $c_c$ ). The critical concentration in volume fraction of polymer,  $v_{2c}$ , multiplied by the chain length  $Z$  ranges from 130 to 280 for PVA and 371 to 1005 for PS, respectively, and possess the same order of magnitude as the value of  $Z_{cV_2}$  obtained from the log-log plots of viscosity against chain length for many systems of polar and non-polar polymers. The product  $c_c r Z^{1/2}$  ( $r$  is the density of the solution) is constant and independent of the chain length. This may be explained well with the equivalent sphere model.

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### Correlations for Non-Newtonian Flow

ROGER S. PORTER and JULIAN F. JOHNSON, *California Research Corporation, Richmond, California.*

Empirical correlations represent an important part of present knowledge of non-Newtonian polymer flow. In this work, several general correlations for non-Newtonian flow are extended and interpreted. Attention is given particularly to concentrated systems of flexible, linear polymers such as polystyrene, polyethylene, and polyethylene glycol. New viscosity-shear studies and extensive literature data are interpreted. Limiting conditions for non-Newtonian flow are discussed in terms of polymer molecular weight, concentration, temperature, and shear. The merits of several forms of reduced variable correlations are discussed. Variations in temperature coefficients and apparent flow activation energies with shear are also appraised.

### The Upper Newtonian Regime in Polymer Solutions

E. W. MERRILL, H. S. MICKLEY, A. RAM, *Department of Chemical Engineering*, and —  
W. H. STOCKMAYER, *Department of Chemistry, Massachusetts Institute of Technology, Cambridge, Massachusetts.*

A synopsis of recent discoveries concerning, or related to, the upper Newtonian regime is given. These include the following.

1. The second Newtonian viscosity (the value found in the upper Newtonian regime) was obtained in a Couette viscometer at shear rates of 60,000 to 100,000  $\text{sec}^{-1}$  for solutions of polystyrene, polyisobutylene, and polymethyl-methacrylate, the variables being polymer molecular weight, polymer concentration, and solvent.

2. The viscosity number  $n_{sp}/c$ , computed from the second Newtonian viscosity was found to increase with polymer concentration  $c$  if the polymer molecular weight  $M$  were less than  $1 \cdot 10^6$ ; to show a *plateau of concentration independence* for  $M$  between  $1 \cdot 10^6$  and

$5 \cdot 10^6$ ; and to decrease with increase of  $c$  for  $M$  greater than  $5 \cdot 10^6$ .

3. In the molecular weight range  $1 \cdot 10^6$  to  $5 \cdot 10^6$ , the "plateau" value of the viscosity number,  $(n_{sp}/c)_p$ , was found to vary with  $M$  according to the relation:  $(n_{sp}/c)_p = K/M^b$ . For good solvents the value of  $b$  was 0.2 to 0.4; for poor solvents  $b$  approached 0.5. Comparison with the Mark-Houwink equation for conventional intrinsic viscosity:  $[\eta] = K M^a$  ( $a = 0.6$  to  $0.8$  for good solvents, decreasing with poor solvents to  $0.5$ ) reveals a symmetry around 0.5 of  $a$  and  $b$ , the significance of which is unknown.

4. For polymers of molecular weight below  $1 \cdot 10^6$ , extrapolation to zero concentration of the viscosity number, based on the second Newtonian viscosity, leads to a value almost identical with the conventional intrinsic viscosity.

5. Under *steady* shear stress of high level (17,000 dynes/cm<sup>2</sup>) there is negligible degradation by molecular scission of polymers up to  $2 \cdot 10^6$  mol. wt. and 2 g/dl. in concentration regardless of duration of shear. However, shaking a bottle of the same polymer, when highly diluted, produces degradation. This suggests that rate of change of shear stress is more important than level of shear stress, in determining molecular scission.

6. The onset of Taylor vortices in the Couette viscometer used, for solutions of all polymers of  $5 \cdot 10^6$  mol. wt. or less, occurred within the same range of Reynolds number as was found for pure solvents, when the second Newtonian viscosity was used in the Reynolds number.

7. Certain Newtonian liquids, including glycol-water, glycerol-water, paraffin oil, and standard calibrating oil, showed non-Newtonian flow (viscosity decreasing) commencing at around 60,000  $\text{sec}^{-1}$  shear rate.

### The Rheology of Poly(neopentyl succinate)

L. J. GARFIELD, S. E. PETRIE, and D. W. VANAS, *Research Laboratories, Eastman Kodak Company, Rochester, New York.*

The Newtonian viscosity-temperature behavior of poly(neopentyl succinate) as well as that of poly(pentamethylene terephthalate-co-pentamethylene isophthalate 90/10) was found to follow rigorously the WLF equation to temperatures well in excess of  $T_g + 100^\circ\text{C}$ . In fact, the experimental data extend to values of the fractional-free volume of 0.13, considerably above the established limit of 0.08. The WLF parameters were found to be:  $f = 0.036$  and  $a_2 = 4.7 \times 10^{-4} \text{ deg}^{-1}$ .

Data on the molecular weight dependence of the melt viscosity of unfractionated samples of poly(neopentyl succinate), taken at an arbitrary, fixed number of degrees above  $T_g$ , and plotted as  $\log \eta$  vs.  $\log [\eta]$ , are satisfactorily represented by two intersecting straight lines. The critical entanglement point, determined from the intersection point of these lines, occurs in the region of a  $M_w$  of 17,000.

The glass transition temperature of this polymer as

a function of molecular weight approaches a limiting value in the region of  $M_c$ . The proposition is made that the limiting value of  $T_g$  is directly related to the average molecular weight between entanglement points.

The response of poly(neopentyl succinate) to variation of the shear rate is typically pseudoplastic. A calculation of the shear gradient molecular weight by the method Bueche and Harding is made from a flow curve and is compared with  $M_w$ .

Tuesday Afternoon, October 31

1:30-4:00

Technical Session. Chairman: DR. C. C. McCABE

Theoretical and Experimental Studies on Temperature Dependence of Mechanical Behavior of Polymer Solids by Creep Recovery Method

(Part 1) Uniaxial Compression Method

KATSUHIKO ITO, *The Institute of Physical and Chemical Research, Komagome, Bunkyo-ku, Tokyo, Japan.*

A simple measuring method for temperature dependence of mechanical behavior of polymer solids is proposed. This method is performed by measuring creep recovery from a room temperature to elevated temperatures. In this paper, the creep recovery method in uniaxial compression is presented theoretically and experimentally. Test specimens used are cast resin of diallyl phthalate homopolymer as a cross-linked polymer and hard polyvinyl chloride resin as a linear polymer. The transition temperature to entropy elasticity and flow temperature can be demonstrated clearly by  $J_c$  (creep compliance in strain recovery) -  $T$  (temperature) curves. Further, the temperature dependence of characteristic relaxation times may be computed from  $J_c$ - $T$  curves and compared with Williams-Landel-Ferry's law.

Two-Dimensional Rheology and Its Application to Cereal Protein Monolayers

NICHOLAS W. TSCHOEGL, *Bread Research Institute of Australia, Epping Highway, North Ryde, N.S.W., Australia.*

The rheology of monolayers, or unimolecular films, often called *two-dimensional*, or *surface rheology* because it considers surface instead of volume moduli, is a relatively new research tool offering some interesting possibilities and advantages.

Some of the surface rheological techniques at present available for the investigation of monolayers at mobile interfaces will be discussed and their use will be illustrated by results obtained in a study of cereal protein films.

Elastic Effects in the Extrusion of Polymer Solutions

D. L. MCINTOSH, *Polychemicals Department, E. I. DuPont de Nemours Company, Wilmington, Delaware,* and —

J. M. MCKELVEY, *Chemical Engineering Department, Washington University, St. Louis, Missouri.*

A theory was formulated to describe the extent of swelling of a viscoelastic liquid emerging from a tube which was based on the concept of elastic strain recovery. Simplified forms of the theoretical expression for the ratio of the radius of the swollen stream to that of the tube were obtained which were applicable to the analysis of experimental data.

In a series of eleven experiments on a 2% aqueous solution of carboxymethyl cellulose, the flow and swelling data required for verification of the theory were obtained. These experiments involved variation in tube geometry with L/D values of 50, 100, 150 and tube diameters of 1.12 and 1.62 millimeters and variation in polymer temperature from 30 to 40°C. Both horizontal and vertical tubes were used.

The flow data obtained were found to be consistent with the rheological principles describing the flow of non-Newtonian, pseudoplastic polymer solutions. It was shown that the swelling data were adequately represented by the simplified theoretical expressions. Two elastic properties of the solution obtained from the fit of theory to data were found to be in good agreement for each of the eleven runs.

The time required for strain recovery was found to be of the order of one to two magnitudes less than that characteristic of the deformation process. An explanation for this difference was given.

Experiments with fluids containing hydroxy groups were described which illustrated the onset of elastic swelling with increase in molecular complexity.

The Adhesion of Acrylate Polymers to Cellophane as Measured by the Peeling Test

J. L. GARDON, *Textile Research Laboratory, Rohm and Haas Co., Philadelphia 37, Pennsylvania.*

Cellophane films were glued together with an acrylic polymer and peeled apart on the Instron tester. At a given adhesive layer thickness the peeling force increased with the peeling rate at low rates and the failure was cohesive. As the rate of deformation increased with increasing peeling rate, the cohesive strength of the binder increased until it surpassed the adhesive bond strength. This happened at high peeling rates and here the force was rate independent and the failure was adhesive. At a given peeling rate the force increased with the adhesive layer thickness because the work expended to deform the binder prior to rupture necessarily increases with the adhesive layer thickness.

The published theories on the peel adhesion test were extended and modified. An expression was derived giving a relationship between the peeling force and the thickness of the binder layer in terms of the adhesive bond strength, moduli of the binder and substrate, thickness of the substrate and width of the sample. The

theory fits the adhesive failure data obtained at high peeling rates very well and the parameters calculated from the experimental data are consistent with estimates based on independently determined rheological properties. The theory also allows the calculation of the magnitude of the rate of strain in the laminate components and of the compressive and tensional forces in the glue line. At 5 in./min. peeling rate the binder is deformed at around  $10^5$  to  $10^6$  %/min, and the substrate deformation rate is  $10^3$  to  $10^4$  %/min. The compressive force in the glue line prior to failure is relatively large, of the same order of magnitude as the measured peeling force at low interlayer thickness. The tensional force is the sum of the compressive and measured force and is roughly proportional to the area in the glue line which is under tension.

### Tuesday Afternoon, October 31

Business Meeting — 4:00-5:00

### Tuesday Evening, October 31

Smoker and Presentation of the Bingham Medal—8 p.m.  
Introduction of Medalist by Dr. John H. Dillon

### Wednesday Morning, November 1

9:00-12:00

*Technical Session. Chairman: DR. J. H. ELLIOTT*

#### Phenomenological Characterization of a Rheopectic Suspension

J. B. YANNAS, *Frick Chemical Laboratory, Princeton University, Princeton, New Jersey*, and —  
R. W. GONZALEZ, *W. R. Grace & Company, Dewey & Almy Chemical Division, Cambridge, Massachusetts*.

The flow behavior of an aqueous suspension at room temperature has been investigated at shear rates up to  $4500 \text{ sec}^{-1}$  in a coaxial cylinder viscometer and has been found to be rheopectic. Shearing stress *vs.* shear rate data ("flow curves") and stress *vs.* time data are presented and it is shown that extraneous effects that might have interfered in the exhibition of rheopexy (such as turbulence, air entrainment and evaporative loss of water) were absent during testing.

During the early stages of shear, the suspension is shown to behave as if it were a linear viscoelastic liquid whose stress-time relation obeys an integrated form of the well-known Maxwell equation. During this early stage, the suspension shows marked hysteresis in flow whereas, after prolonged shear, the flow behavior becomes time-independent and quite clearly Newtonian.

A definition for rheopectic flow which extends the traditionally accepted one is proposed, and the existence of this type of flow at the relatively high shear rates of  $1520$ - $2250 \text{ sec}^{-1}$  (as demonstrated in this work) is discussed in the context of contrasting views on rheopexy.

#### The Flow of Dilute Suspensions Through Cylindrical Tubes

H. L. GOLDSMITH and S. G. MASON, *Department of Chemistry, McGill University, Montreal, Canada*.

The motions of single rigid spheres, rods and discs and of fluid drops suspended in liquids undergoing Poiseuille flow have been studied.

With the exception of effects due to interaction with the wall and neglect of particle size, the angular rotation of the spheres, rods and discs were found to be in good agreement with the theory of Jeffery provided that the equivalent axis ratio  $r_e$  was used instead of the measured axis ratio  $r_p$  of rods and discs.

The periods of rotation of the rigid particles near the wall of the tubes were greater than those calculated from the above theory. In the case of spheres the observed periods of rotation were in good agreement with those calculated from the theory of Vand for rotation in proximity to a wall.

The deformation and burst of liquid drops were similar to those observed previously in Couette flow. At low deformation excellent agreement with theoretical equations due to Taylor and Cerf was found.

The existence of axial migration for deformable drops and its absence in the case of rigid particles was demonstrated. Good agreement with an approximate theory of migration based on the radial variation of the velocity gradient in Poiseuille flow was obtained.

The relevance of these phenomena to the flow of blood in living systems is discussed.

#### Viscosity of Concentrated Suspensions of Rigid Particles\*

STEPHEN PRAGER, *Department of Chemistry, University of Minnesota, Minneapolis, Minnesota*.

The principle of minimum energy dissipation has been applied to estimate viscosities in concentrated suspensions of rigid particles. Rigorous lower bounds on the viscosity can be obtained in terms of a function  $G(r_1, r_2, r_3)$ , defined as the probability that three points forming a triangle with sides  $r_1, r_2$ , and  $r_3$  will, when placed at random in the suspension, all be found to lie in regions occupied by the suspending fluid. A particularly simple inequality appears as a special case: for an isotropic suspension,

$$\frac{n_0}{n} < (1 - v_s) \left(1 - \frac{3}{5}v_s\right),$$

where  $n$  is the viscosity of the suspension,  $n_0$  that of the suspending fluid, and  $v_s$  the volume fraction of suspended material; this inequality is valid at all concentrations and for all shapes of suspended particles.

\*This work is being supported by the Air Force Office of Scientific Research.

#### The Viscosity of Liquid Helium II

A. C. HOLLIS HALLETT, *Department of Physics, University of Toronto, Toronto 5, Ontario, Canada*.

Measurements have been made of the viscosity of the normal component of liquid Helium II between  $2.17^\circ\text{K}$

and 0.79°K using a Couette Viscometer in which the inner cylinder is stationary while the outer rotates. The viscosity is found to have a value of about 24 micro-poise at 2.17°K which falls to a value of about 12.8 micro-poise at 1.82°K. It then rises with decreasing temperature, slowly at first, then more rapidly below 1.2°K to a value of about 173 micropoise at 0.79°K. The variation of the viscosity with temperature below 1.8°K is explained by the detailed theory of Landau and Khalatnikov and a simple description of this theory will be given.

#### Plastic Flow of Aluminum Single Crystals as Revealed by Interferometric Cinemicrography

R. B. POND, *Department of Mechanics, The Johns Hopkins University, Baltimore, Maryland.*

The phenomenon of plastic deformation by the slip process is illustrated by means of a technique combining multiple beam interferometry and motion photomicrography during tensile and/or compressive tests. Data is presented from sixty tests showing the variation of the slip velocities resolved in the slip direction as a function of strain rate, crystal orientation, and frequency of co-active slip bands. A theory is proposed showing the correlation of the elastic energy stored prior to plastic deformation and the velocity of slip. This theory is supported by the data from the experiments. Data showing the effect of reversed stress in the plastic region in producing microscopic extrusions and notches is presented. The time dependence of strain hardening is illustrated and the ability to deslip aluminum with reversed stresses is experimentally demonstrated. (Motion picture documentation of this experiment is presented.) From the afore mentioned proposed theory a two-pronged approach for developing the activation energy for slip is postulated. The experiments, analysis and results of these approaches are presented and the results compared with those derived by conventional methods.

#### Vibrations and Wave Propagation in Aluminum Alloys at Elevated Temperatures

S. R. BODNER and A. F. FRASER, *Division of Engineering, Brown University, Providence, Rhode Island.*

Measurements of the dynamic modulus and damping of commercially pure aluminum (1100) and an aluminum alloy (2024T4) have been obtained over a frequency range of 20-3500 cycles/sec. at temperatures from 80°F to 1100°F. For the small test oscillations the results were found to be amplitude independent. The frequency and temperature dependence of the results is discussed in relation to linear viscoelastic representation, metallurgical properties, and possible temperature-time correspondence. Some pulse propagation tests were performed but their analysis is complicated by the occurrence of plastic waves at high temperature.

Wednesday Afternoon, November 1

(two simultaneous sessions)

*Technical Session on Application. Chairman: DR. E. A. COLLINS*

SESSION I — 1:30-3:30 at the Auditorium  
Wisconsin Center

#### Melt Viscosity — Viscosity Average Molecular Weight Relations of Polyethylenes

R. W. LONGWORTH, W. F. BUSSE, *Polychemicals Department, E. I. du Pont de Nemours & Co., Wilmington, Delaware.*

The Newtonian melt viscosity of various polyethylene-paraffin wax mixtures has been found to vary with about the 4.5 to 12th power of the weight average molecular weight,  $M_w$ , of the mixtures, rather than with the expected 3.4 power. However, the viscosities vary with the 3.4 power of the viscosity average molecular weight,  $M_v$ , for mixtures having viscosities from about  $10$  to  $10^7$  poises at 150°C. The exponent  $a$  in the relation  $M_v = (S w_i M_i^a)^{1/a}$  varied from about 0.75 to 0.8 for high density polymers down to 0.4 or 0.3 for low density resins. The relations between the values of  $a$  and long chain branching, and the effects of  $a$  on non-Newtonian flow properties, were explored. The theory used in calculating  $a$  assumes that the melt viscosity of sharp fractions varies with the 3.4 power of the molecular weight. If other relations are assumed, different values of  $a$  would be obtained. The effect of arbitrary systematic variations in  $a$  on the viscosity —  $M_v$  relations is explored, and some implications of these data for the current theories of melt viscosity are discussed.

#### The Dynamics of a Viscoelastic Particle Between Sliding Surfaces

W. D. MAY, *Wood & Fibre Physics Division, Pulp & Paper Research Institute of Canada, Montreal.*

In an attrition process studied by the author, viscoelastic particles, usually in the form of cylinders, are formed between lubricated sliding surfaces. One of these surfaces has projections on it which causes the viscoelastic cylinder to roll. The amount of slipping between this driving surface and the cylinder has been studied theoretically. It has been shown that the amount of slipping depends on the relaxation time of the viscoelastic material, on the ratio of projection size to cylinder radius, and on the velocity of the driving surface.

Two modes of slipping occur, depending on the ratio of projection size to cylinder radius. In one mode, as the velocity of the driving surface increases, the cylinder velocity increases also. The amount of slip, however, increases up to a certain velocity and then decreases again for further increases in the driving plate velocity.

In the other mode, as the velocity of the driving plate increases up to a threshold velocity, the cylinder velocity

and the amount of slip both increase. Beyond the threshold velocity, the amount of slip increases rapidly, and the velocity of the cylinder suddenly decreases and continues to decrease for increasing driving plate velocity.

It is suggested that these results may be of general value in studying the movement of viscoelastic wear fragments in sliding bearings of a hard material on a viscoelastic one.

## Rheological Studies with the Brabender Farinograph

I. HLYNKA, *Grain Research Laboratory, Board of Grain Commissioners for Canada, Winnipeg 2, Manitoba, Canada.*

The Brabender farinograph (plastograph) is a widely used recording mixer. For the rheological system flour-water (dough) the linear relationship between farinograph absorption and mobility has proved useful in a number of rheological studies. Results show that increase in temperature or in amount of added salt increase the mobility indicating that an increase in either of these factors increases the amount of water of mobility. Increasing the rate of shear (mixing speed) brings about a greater dependence of the mixing process on the shearing force and a lesser dependence on the "lubricating" effect of the water of mobility.

The activation energy for dough mobility is of the order of that for viscous flow of water, suggesting that water plays a major role in dough mobility.

In common with some rheological systems, dough conforms to the so-called power law. The logarithm of the rate of shear varies linearly with shear stress or with variables which are a direct function of shear stress. The shear stress vs. square root of shear rate relationship, used in the literature for some systems, is linear for dough.

The effect of temperature, salt and rate of shear on development time (time to minimum mobility) has been examined also. Linear relationships were obtained for development time vs. temperature, development time vs. salt concentration, and development time vs. reciprocal of rate of shear.

## The Splitting of Thin Liquid Films II. Cavitation Dynamics

RAY D. HOFFMAN and RAYMOND R. MYERS, *Lehigh University, Bethlehem, Pennsylvania.*

The previous paper in this series showed photographically that cavitation occurs in the roll application of films and provided a means of calculating the tack impulse from the cavity population. The velocity-viscosity ( $Un$ ) product controlled the cavity population.

A supplementary correlation is provided from analysis of cinematographic sequences of film splitting from a vantage point inside the roll. In the new correlation the growth rate of individual cavities is used to delineate the forces involved in splitting, and a hypothesis is presented that cavity growth is a function of the pressure differential between the inside and the outside

of the cavity, the viscosity-rate of growth ( $nv$ ) product, and the surface tension.

Experimental verification of certain deductions from the hypothesis was obtained from studies of the rate of cavity growth ( $v$ ) at various selected levels of the controllable  $Un$  parameter.

## Analysis of Forces Causing Flow in Roll Coaters

G. B. SCHNEIDER, *Minnesota Mining and Manufacturing Co., St. Paul, Minnesota.*

The Banks and Mill Equation for the relation of the variables in a roll coater,

$$\frac{Pc^2}{12uU \sqrt{2rc}} =$$

$C + \frac{1}{2} a + \frac{1}{4} \sin a - K_1 (\frac{3}{8} a + \frac{1}{4} \sin 2a + \frac{1}{32} \sin 4a)$  was solved for  $K_1$  over a selected range of machine variables using an I.B.M. 705 Computer. In this case boundary conditions for a non-submerged, non-symmetric system were used and the  $K_1$  value found to be constant within  $\pm 5\frac{1}{2}\%$  over the selected range. Relating this to the flow rate through the coater nip, the wet film thickness on the coating roll was established as

$$t = K_1 c^{1/2}$$

A two roll experimental coater was then used to obtain confirming data by physical measurement of flow rate through the coater nip.

Motion pictures were also made and interpreted to obtain visual evidence of the behavior predicted by the equations.

## Wednesday Afternoon, November 1

*Technical Session on Mechanics of Continua.*

*Chairman: DR. H. MARKOVITZ*

SESSION II — 1:30-4:30 in Room 210

Wisconsin Center

### Mechanical Behavior of Rubbery Material

P. J. BLATZ and W. L. KO, *California Institute of Technology, Pasadena, California.*

An experimental investigation has been carried out on continuum SBR — rubber and polyurethane foam rubber. Data from both uniaxial and biaxial tensile tests are presented and discussed. From these data the relation of the strain energy function  $W$ , to the invariants  $I_1, I_2, I_3$  of the deformation tensor can be determined. In particular the results of the tests show that:

(1)  $\frac{dW}{dI_1}$  and  $\frac{dW}{d(\frac{I_2}{I_3})}$  are both positive constants for continuum rubber and foam.

(2)  $\frac{dW}{dI_3}$  is zero for continuum rubber, positive for foam.

This statement is interpreted in terms of an effective Poisson's ratio which holds for large strains.

## The Viscous Heating Correction for Viscometer Flows

ELLIOT A. KEARSLEY, *National Bureau of Standards,  
Washington, D. C.*

At very high rates of shear, many liquids show a decrease in apparent viscosity. It is often not clear whether this effect is due to non-Newtonianism or to viscous heating. An obvious approach to the problem is to study the flow for a Newtonian liquid with a reasonable viscosity-temperature dependence and to see if the apparent viscosity decrease can be attributed to viscous heating. This approach involves, in general, the simultaneous solution of the Navier-Stokes equations and the energy balance equation, which are coupled through the viscosity-temperature relation. It is not sufficient to use a linear energy-temperature relation, because the solution depends very strongly on the coupling which for realistic liquids is strongly non-linear. For certain particular flow geometries, including Couette flow and infinite pipe flow, the equations can be uncoupled and result in a set of non-linear ordinary differential equations. Furthermore, if an exponential viscosity-temperature law is assumed, approximating that of many real liquids, the differential equations can be integrated in closed form in terms of surprisingly simple functions. The solutions exhibit an interesting double-valuedness typical of non-linear equations.

## Conditions for Second Order Waves in Hypo-Elastic Materials

BARRY BERNSTEIN, *Naval Research Laboratory, Wash-  
ington, D. C.*

The conditions are obtained for a second order discontinuity in displacement across a propagating surface, or wave surface, in a hypoelastic material. The discontinuity is shown to be determined by a vector which must be a proper vector of a certain matrix, the acoustic matrix. This matrix depends upon the stress and the normal to the surface. The square of the speed of propagation, or wave speed of the wave surface, must be a proper value of the acoustic matrix. The condition that the acoustic axes always be real and orthogonal and the square of the wave speeds real is obtained. This condition is the same as the criterion that a Cauchy-elastic hypo-elastic material possess a strain energy. But there exist non-elastic hypo-elastic materials for which the acoustic axes are always real and orthogonal and the square of the wave speeds are real.

## Orientation Induced by Flow

J. L. ERICKSEN, *The Johns Hopkins University, Balti-  
more, Maryland.*

We use a relatively simple theory of anisotropic or orientable fluids to determine conditions under which homogeneous motions are likely to produce a high degree of orientation of structure present in the fluid.

## On "Substantially Stagnant Flows," a Kinematical Concept Useful in Investigations of the Physics of Viscoelastic Fluids

BERNARD D. COLEMAN, *Mellon Institute, Pittsburgh,  
Pennsylvania.*

Consider flows in which each particle, as it moves along its flow line, always sees behind itself essentially the same past deformation history. (Here I use the words "essentially the same" to indicate that I do not distinguish between two histories which differ by a superimposed rigid rotation.) Let us call such a flow a *substantially stagnant flow*. Simple shearing flow, torsional flow, Poiseuille flow, and helical flow (and thus Couette flow and axial flow between concentric pipes) are all substantially stagnant. Kinematical theorems will be presented here which give simple criteria for determining when a given substantially stagnant flow is equivalent (as far as the constitutive equations are concerned) to simple shearing flow. Thermodynamical arguments will be presented which show that, in the case of a simple fluid, a substantially stagnant flow for which the internal energy density is constant along a flow line has the following properties: (a) the temperature and entropy density are also constant along each flow line; (b) the dissipation of power is non-negative everywhere in the fluid.

## On Streaming Birefringence

BERNARD D. COLEMAN, *Mellon Institute, Pittsburgh,  
Pennsylvania,* and —  
RICHARD TOUPIN, *U. S. Naval Research Laboratory,  
Washington, D. C.*

We consider the general phenomenological theory of materials for which the stress and the effective local electric field depend on the history of the deformation gradient and the electric polarization. The theory is used to find the limitations placed by material objectivity and material isotropy on the possible relations between the stress and the birefringence in a flowing viscoelastic fluid.

## SOCIETY NOTICES

### West Coast Regional Meeting of the Society of Rheology

A West Coast Regional Meeting of The Society of Rheology will be held on February 1-2, 1962 at the California Institute of Technology in Pasadena. Plans for the meeting include four technical sessions and a dinner and social gathering on Thursday evening, February 1.

Papers are now being solicited for the meeting and prospective speakers are invited to contact Thor L. Smith, Stanford Research Institute, Menlo Park, California. Titles are due on November 15 and 200-word abstracts by December 10, 1961. Abstracts will be duplicated prior to the meeting and mailed to participants and others upon request. Interested persons

should contact Dr. Smith who is arranging the technical program. In addition to contributed papers, several review talks by well-known persons are being arranged.

Local arrangements for the meeting are being made by Paul J. Blatz, Guggenheim Aeronautical Laboratory, California Institute of Technology, Pasadena, California. To provide overnight accommodations, rooms have been reserved at the Saga Motel, 1633 East Colorado Avenue, Pasadena. Other plans include luncheons at the Cal Tech Athenaeum and courtesy coffee breaks. Persons desiring room reservations and preregistration details about the meeting should contact Dr. Blatz.

#### Annual Meeting, 1962

The Johns Hopkins University, Baltimore, Maryland has extended an invitation for our Annual Meeting for next year. This has been accepted, and the Meeting will be held at Johns Hopkins, October 29, 30 and 31, 1962 under the local direction of Professor J. L. Ericksen.

#### Visits by Rheologists from Abroad

Members of the Society of Rheology are invited to submit names of foreign rheologists or physicists whom they would like to invite for brief visits to their institutions in this country under the visiting foreign scientists program of the American Institute of Physics and the American Association of Physics Teachers. The A.I.P. funds provide for the visitor's travel expenses to this country and for an honorarium to the visitor. The host or his institution will be expected to provide for the visitor's room and board during his visit at a particular location and to contribute between \$50 and about \$200 to the A.I.P. for his travelling and other expenses while he is in the U. S. It is expected that each visitor will visit about half a dozen institutions during such a trip, spending between a day and a week at each institution. One, or possibly two rheologists may be accommodated under this program during each of the years beginning in September 1961 and September 1962.

The Society of Rheology representative on the A.I.P. educational advisory committee is Professor A. B. Metzner, University of Delaware, Newark, Del. Suggestions should be sent to him for compilation and forwarding to A.I.P. Such a letter should also indicate the desired duration of the visit and give some indication of the willingness of the host's institution to assume the above financial obligation, although a firm commitment is not yet required.

#### Society Representation

John Elliott has appointed John P. Tordella, of E. I. du Pont de Nemours and Company, Inc., Experimental Station, Wilmington, Delaware to be the representative of the Society for the Council of the American Associa-

tion for the Advancement of Science. Your Editor has been appointed to represent the Society on the U. S. National Committee of the International Union of Theoretical and Applied Mechanics.

#### Letter Ballot for Election of Officers

Don't forget your letter ballots for new officers. They must be sent in the envelope provided with the ballot form, signed, and postmarked no later than October 1, 1961.

## FOURTH INTERNATIONAL CONGRESS ON RHEOLOGY 1963

The Fourth International Congress on Rheology will take place from August 26 to August 30, 1963 at Brown University, Providence, Rhode Island, U. S. A., under the auspices of the International Committee on Rheology with the (U. S.) Society of Rheology as the host society. Rheologists who wish to present papers at the Congress are invited to submit, by March 30, 1963, abstracts of not more than 500 words, in a form suitable for publication, to

PROFESSOR E. H. LEE  
EDITOR OF THE PROCEEDINGS OF THE  
FOURTH INTERNATIONAL CONGRESS ON RHEOLOGY  
BROWN UNIVERSITY  
PROVIDENCE 12, R. I., U. S. A.

It is intended that twenty minutes will be allotted for the presentation and ten minutes for the discussion of each contributed paper.

In addition to contributed papers, the program of the Congress will include lectures of longer duration on fields of rheology of especial current interest and social occasions on which rheologists from the various countries represented may exchange views.

Accommodations for those attending the Congress and their spouses, consisting of single or double rooms, will be available in the dormitories of Brown University. Alternatively, some attendants may prefer to stay in one or the other of the hotels in the Providence area. Meals will also be available in the University. Information regarding the price of accommodations and meals will be given in later announcements.

Dr. R. S. Marvin and Professor R. S. Rivlin are Co-Chairmen of the Congress. All enquiries regarding it should be addressed to:

PROFESSOR R. S. RIVLIN  
CO-CHAIRMAN, FOURTH INTERNATIONAL CONGRESS  
ON RHEOLOGY  
BROWN UNIVERSITY  
PROVIDENCE 12, R. I., U. S. A.

